The Joint Behavior of Sovereign CDS Spreads and Country Equity Risk: an Empirical Analysis

Patrizia Stucchi

Abstract Starting from the Merton [14] structural model it is possible to show that an inverse relationship holds between firms CDS spreads and their equity premium (Friewald et al. [7]). My work investigates empirically if the relation holds also for countries. To this aim, I have considered the daily CDS spreads on the 5 years government bonds of a set of countries (Brazil, China, Greece, Italy, Russia and US) and compared year by year their behavior with that of the countries daily equity premium defined in terms of Conditional Value at Risk (CVaR) over the period 2006-2012. The results confirm a strong inverse relationship between CDS spreads and equity risk premium.

Keywords Country Risk Country Equity Conditional Value at Risk Sovereign Credit Default Swap Risk Measures

JEL G32 JEL G01 JEL F30 JEL C63

1. Introduction

Friewald et al. [7] show that risk premia in equity and credit markets are strongly related. They start from the Merton [14] structural model and obtain an inverse relationship between firms CDS spreads and their equity premium. The main purpose of my work is to investigate empirically if there is an analogous relation between sovereign risk and country equity risk premium. Sovereign risk may be measured in terms of credit default swaps on government bonds, while country equity risk in terms of Conditional Value at Risk (CVaR) of equity market indices. In my paper there is the analysis of the joint behavior of sovereign Credit Default Swaps (CDS) spreads and of risk premium on countries’ equity indices. The basic idea is that sovereign CDS spreads represent the insurance cost against country’s default for bondholders, that is
spreads may be intended as a negative risk premium. On the other hand, equity market investors require a positive equity risk premium. In a simplified market with two asset classes only, sovereign bonds and equities, if country risk rises, investors should sell sovereign bonds and invest on stocks and vice versa. Equivalently, increasing sovereign risk requires higher insurance cost, that is higher CDS spreads: investors aiming to reduce their risks or insurance costs should shift to equity markets and this implies equity premium reduction. This means that an inverse relationship between CDS spreads and equity risk premium should hold, as for firms in the theoretical structural framework due to Merton [14].

In the empirical investigation, the daily CDS spreads on the 5 years government bonds of a set of countries (Brazil, China, Greece, Italy, Russia and US) have been considered. Their year by year behavior have been compared with that of the countries daily equity premium defined in terms of Conditional Value at Risk (CVaR) over the period 2006-2012. The results of the linear regression of country equity premium over CDS spreads differ from one country to another, but in some years they confirm a very strong inverse relationship between CDS spreads and risk equity premium.

The paper is organized as follows. In Section 2 there is a short overview of the theoretical insights about the links between market and credit risk. Section 3 describes the structural model by Merton [14] and its application to sovereign credit risk modeling analyzed by Friewald et al. [7]. Section 4 is concerned with the methodology of the empirical analysis. In Section 5 the results obtained have been shown and analyzed. Section 6 contains the conclusions.

2. Theoretical framework

In 2000 Jarrow and Turnbull [11] observed that practitioners and regulators often calculate credit risk and market risk separately and then simply add their values to obtain a global risk measure. They claim that this unsatisfactory practice is based on the difficulty to estimate the correlation between market and credit risk and that this is equivalent to assume perfect positive correlation. Many authors have considered this problem, but it remains unsolved and regulators themselves are well aware of the “puzzle”; in fact, they say in 2009 that “for many reasons, both historical and practical, market and credit risks have often been treated as if they are unrelated sources of risk: the risk types have been measured separately, managed separately, and economic capital against each risk type has been assessed separately.” (Basel Committee [2]).

Credit and market risk are strictly related and their global influence cannot be evaluated by simple addition. In particular, considering a single firm in the stylized Merton model [14] (structural form approach), the equity is a call option on the firm value (assets) with the entire debt as strike price. In this theoretical framework, it is possible to show that the equity premium is exactly the opposite of the spread of a single name CDS written on the debt (Friewald et al. [7]). This result may be derived substantially from the put-call parity relationship, thinking to the CDS spread in terms of a long position in a put option on the assets with the debt as exercise price. Implementing the structural approach implies significant practical difficulties due to the lack of observable market data on the firms value and these difficulties become greater dealing with sovereign data. Merton et al. [6] applied the structural model to sovereign risk with a CCA (Contingent Claim Analysis) approach based on the structural model, but their method requires the estimation of sovereign balance sheet data. In an older paper, Jarrow and Turnbull [9,10] overcome these difficulties, suggesting a reduced form approach and inferring the conditional martingale probabilities of default from the term structure of credit spreads.
3 The structural approach and credit risk premia

This section describes the Black, Scholes [5] and Merton [14] structural approach. Merton considers a firm with a simple capital structure, that is the value of the firms’ assets \( V(t) \) is given by the value of equity \( E(t) \) and the value of risky debt \( v(t;T) \) corresponding to the present value of a zero-coupon bond with a face value \( D \) and maturity \( T \) subject to the firm’s risk of default:

\[
V(t) = E(t) + v(t;T) \tag{1}
\]

At maturity, if the value of the firm’s assets \( V(T) \) is greater than the amount owed to the debt holders (the face amount \( D \)) then the equity holders repay the bondholders and retain the firm. If the value of the firm’s assets is less than the face value, the firm goes bankruptcy. In this case, if there are no costs associated with default, bondholders take over the firm and the value of equity becomes zero, assuming limited liability. In this simple framework, if \( V(t) \) and \( E(t) \) follow a geometric brownian motion, using Black and Scholes [5] arguments in presence of a risk free rate \( r \), Merton [14] shows that the value of equity is the value of a European call option on the firm value \( V(t) \) with strike price the face value of debt \( D \):

\[
E(t) = c[V(t), D, t, T, r] = c_t \tag{2}
\]

Therefore, the value of risky debt \( v(t;T) \) may be rewritten from condition 1 in the following way:

\[
v(t;T) = V(t) - c_t
\]

that is, the risky debt value is the firm’s asset value less the value of the described call. On the other hand, using put-call parity, it is:

\[
v(t;T) = D \cdot e^{-r \cdot (T-t)} - p_t \tag{3}
\]

again, obviously:

\[
D \cdot e^{-r \cdot (T-t)} = v(t;T) + p_t \tag{4}
\]

The last condition means that bondholders could protect themselves from firm’s default risk entering into a long position in the put option (Friewald et al. [7]). This also means that a long put on the firm’s asset value with strike \( D \) represents a credit protection contract. Hence, the value of the put option is linked to that of a single-name Credit Default Swap (CDS) written on the firm’s defaultable bond. The CDS contract offers credit insurance to the protection buyer (the bondholder) by paying off the loss given default (that is the unrecoverable amount of the face value of the bond). Usually, the protection buyer has to make periodic payments (the CDS spread) to the protection seller until default occurs or until the contract expires; in this case the spread \( s \) can be found from the condition:

\[
\text{Premium Leg (or Protection Leg) } = s \cdot D \cdot \sum_{t=1}^{T} P S_{0,t} e^{-rt} = \text{Default Leg } = (1 - R) \cdot D \cdot \sum_{t=1}^{T} P D_{0,t} e^{-rt}
\]
where \( PS_{0,t} \) is the probability of survival until time \( t \); \( PD_{t-1,t} e^{-r} \) is the probability of default between time \( t-1 \) and \( t \); \( R \) is the recovery rate (and \( 1 - R \) is the unrecoverable part of debt). Assuming that \( R = 0 \); that is all debt is unrecoverable, since default can only occur at time \( T \) in the Merton framework, the CDS contract has the same present value as the put option, therefore, it is:

\[
s \cdot D \left( \sum_{i=1}^{T-1} e^{-rT} + PS_{0,t} e^{-rT} \right) = p_t
\]

With continuous payments, the CDS spread is:

\[
s = \frac{r}{1 - e^{-r(T-t)}} \cdot P_t
\]

Putting together the fundamental equations 2 and 5, it is possible to say that the firm’s equity value is equal to the value of a call option while the CDS spread covering risk on the firm’s debt is equal to a positive constant time the value of the corresponding put option.

In the Black, Scholes [5] and Merton [14] framework, the assumptions imply the existence of a well defined market price of risk for the call and put options. Denoting with \( \lambda_c \) the call premium and with \( \lambda_p \) the corresponding put premium, it is possible to show that (see Appendix A and Friewald et al. [7]) that:

\[
\lambda_c = \lambda_p
\]

The last condition together with equation 5 mean that the equity premium and the CDS premium are linearly and inversely related.

4. Numerical analysis

Starting from the theoretical relationship between firms’ equity and CDS premium, I investigate empirically if it holds also for countries. The idea is that financial institutions may invest in countries equity indices or government bonds. The risk premium on equities should be inversely related with CDS premium if the theoretical relationship holds for countries as for firms.

4.1 The data

The data set consists in the historical daily prices (source: Bloomberg [4]) of the stock indices, risk free rates indices and CDS spreads reported in Table 1:

<table>
<thead>
<tr>
<th>Country</th>
<th>Stock Index</th>
<th>Risk free rate Index</th>
<th>CDS SR 5Y</th>
<th>Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brazil</td>
<td>IBOV</td>
<td>BZAC2000 (Brazil CETIP)</td>
<td>Brazil CDS USD</td>
<td>2006-2012</td>
</tr>
<tr>
<td>China</td>
<td>SHCOMP</td>
<td>SHIF1Y (Shibor 1Y)</td>
<td>ChinaGov CDS USD</td>
<td>2006-2012</td>
</tr>
<tr>
<td>Greece</td>
<td>ASE</td>
<td>EURO12M (Euribor 1Y)</td>
<td>Greece CDS USD</td>
<td>2002-2012</td>
</tr>
<tr>
<td>Italy</td>
<td>FTSE Mib</td>
<td>EURO12M (Euribor 1Y)</td>
<td>Italy CDS USD</td>
<td>2003-2012</td>
</tr>
<tr>
<td>Russia</td>
<td>RTSI$</td>
<td>RRDR1 (Russia Interbank 1Y)</td>
<td>Russia CDS USD</td>
<td>2006-2012</td>
</tr>
</tbody>
</table>

Table 1
The choice of CDS on 5Y government bonds is due to the liquidity of these instruments. With regard to US data, unfortunately, the USD SR 5Y Corp CDS data set started from 2009 and ZTCO CDS EUR SR 5Y (used here) started from 2008 only. On the other hand, Greece and Italy CDS spread were quoted, respectively, from 2002 and 2003, therefore a larger time horizon has been considered for these two countries.

4.2 The countries equity premium and CDS spread

A roll-over mechanism has been adopted in order to obtain daily values for the annual equity premium. Usually, in the literature the equity premium is a reward to risk ratio. The pioneristic reward to risk ratio is the famous Sharpe ratio, that is the expected excess return over the return standard deviation. The evolution of risk measurement theory and the development of coherent risk measures (see Artzner et al. [1], Embrechts et al. [8]) have important implication on risk premium evaluation. Here, I have used a modern reward-risk ratio adopting the return \textit{Conditional Value at Risk (CVaR)} as risk measure. This makes it possible to take into account for the non-normality of returns (for a survey on risk adjusted performance measure see e.g. [16]). In the numerical analysis, formally, the empirical countries equity premium $\lambda^e_t$ is measured as follows:

$$\lambda^e_t = \frac{\mu^e_t - r^e_t}{CVaR^e_t}$$

- $\mu^e_t$ is the stock index return annualized mean of the 250 daily observations preceding $t$;
- $r^e_t$ is the annualized risk free rate index mean of the 250 daily observations preceding $t$;
- $CVaR^e_t$ is the Conditional Value at Risk evaluated at time $t$ with the Johnson [12, 13] approach using the data set of 250 observations preceding $t$. The next Section 4.3 is devoted to $CVaR$ and the Johnson Systems (for further details see Stucchi, Dominese [17]).

Last, the CDS empirical premium is measured exactly by the value of the observed CDS spread at time $t$.

4.3 Johnson Systems and Conditional Value at Risk

The Johnson [12] [13] framework is based on the idea that mean, variance, skewness and kurtosis may characterize sufficiently well the distributions of a wide set of random ariables $X$ with unknown distribution. Johnson claims that any $X$ in this set may be well approximated in terms of a standard normal variable $N$ by a function of the following kind:

$$X = A + B \cdot h \left( \frac{N - C}{D} \right)$$

where $A, B, C, D$ are parameters obtained in terms of the four characterizing indices of $X$. The function $h(\cdot)$ is a non-decreasing monotonic function independent by the variable’s moments (with positive $B$ and $D$). The choice of the type of function $h(\cdot)$ is linked to the values of the skewness and kurtosis indices.
The functional form $h(\cdot)$ suggested by Johnson are summarized in the next Table 2:

<table>
<thead>
<tr>
<th>Transformation</th>
<th>$h(\cdot)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log-normal $S_L$</td>
<td>$\exp\left(\frac{N-C}{D}\right)$</td>
</tr>
<tr>
<td>Bounded $S_B$</td>
<td>$\left[1+\exp\left(-\frac{N-C}{D}\right)\right]^{-1}$</td>
</tr>
<tr>
<td>Unbounded $S_U$</td>
<td>$\sinh\left(\frac{N-C}{D}\right)$</td>
</tr>
</tbody>
</table>

Table 2

Following the Johnson approach, it is possible to find closed or quasi-closed formulas for Value at Risk ($V aR$) and $CV aR$ once found the kind of transformation and the parameters in the Johnson framework (see Simonato [15]).

$V aR$ may be defined as the opposite of the $\alpha$– quantile of the return distribution with a fixed probability $\alpha$, that is (e.g. see Embrechts et al. [8]):

$$V aR = \inf \{ x \in \mathbb{R} | F(x) \geq \alpha \} \quad 0 < \alpha < 1$$

and $CV aR$, with reference to the random return $X$, is the opposite of the expected mean of the return restricted to returns value below $-V aR$, that is:

$$CV aR = -E(X/X < -V aR)$$

Both $V aR$ and $CV aR$ can be expressed in the Johnson framework in terms of the $\alpha$– quantile of a standard normal variable, that is in terms of:

$$z = N^{-1}(\alpha)$$

where $N^{-1}(\cdot)$ is the inverse of the cumulative distribution function of the standard normal variable. In all numerical applications, coherently with the main guidelines provided by regulators (see Basel Committee [2]), I have set to 1% the probability and this gives $z = -2:33$.

The next Table 3 summarizes the formulas obtained using Johnson Systems:

<table>
<thead>
<tr>
<th>Transformation</th>
<th>VaR</th>
<th>$CV aR$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_L$</td>
<td>$\exp\left(\frac{z-C}{D}\right)$</td>
<td>$\frac{\mu}{\alpha}N(z-s)$</td>
</tr>
<tr>
<td>$S_B$</td>
<td>$A+\frac{B}{1+\exp\left(-\frac{z-C}{D}\right)}$</td>
<td>$A+\frac{B}{\alpha}\left[\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\frac{-z}{\sqrt{2}}/\alpha} \exp\left(-\frac{u^2}{2}\right) du\right]$</td>
</tr>
<tr>
<td>$S_U$</td>
<td>$A+B\sinh\left(\frac{N-C}{D}\right)$</td>
<td>$A+\frac{B}{2\alpha}\left[e^{\frac{z-C}{D}}N(z-\frac{1}{D})-e^{\frac{-z-C}{D}}N(z+\frac{1}{D})\right]$</td>
</tr>
</tbody>
</table>

Table 3
In the log-normal Johnson transformation SL the parameter A value is zero and that of $B$ is $1$ and $VaR$ may be expressed in terms of the estimated parameters $\mu = E(X)$ and $s$, standard deviation of the normal variable corresponding to $ln(X)$. The log-normal $CVaR$ depends on the estimated mean of $X$, on the fixed level of probability and on the above cited parameter $s$. In the bounded transformation $VaR$ is given by a closed formula while $CVaR$ must be evaluated using numerical procedures, while in the unbounded system there are closed formulas for both $VaR$ and $CVaR$.

4.4 Main statistics

The evaluation of daily $CVaR$ requires daily estimation of the parameters involved in the Johnson Systems, that is mean, variance, skewness and kurtosis of the market stock indices log-returns. The estimation of historical return parameters has been made using the basic standard definitions, that is:

\[
\text{Mean} = \bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i
\]

\[
\text{Variance} = \sigma^2 = \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^2
\]

\[
\text{Skewness} = \frac{N}{(N-1)(N-2)\bar{x}^3} \sum_{i=1}^{N} (x_i - \bar{x})^3
\]

\[
\text{Kurtosis} = \frac{N(N-1)}{(N-2)(N-3)\bar{x}^4} \sum_{i=1}^{N} (x_i - \bar{x})^4
\]

Daily $CVaR$ values have been obtained with a fixed probability level $\alpha = 1\%$ and with a roll-over mechanism over the period 2002-2012 for Greece, 2003-2012 for Italy, 2008-2012 for US and 2006-2012 for the other countries.

5. Results

The comparison of countries equity risk premium and CDS spreads over the whole period does not give in general satisfactory results. The next Tables 4-9 summarize the result of the linear regression of country equity premium over the CDS spread. Nevertheless, considering the premia year by year gives very interesting results in many years (different years for different countries), confirming the negative linear relationship with strong values of $R^2$. The next Tables 4-9 summarize the main results. At the end there are two graphical meaningful examples relative to the behavior of the Chinese equity premium and CDS spread in 2009 and of the Italian equity premium and CDS spread in 2011.
### Brazil

<table>
<thead>
<tr>
<th>Year</th>
<th>Equation</th>
<th>$R^2$</th>
<th>China</th>
<th>Equation</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2007</td>
<td>$y = -0.0317x + 0.0097$</td>
<td>3.14%</td>
<td>$y = -0.0409x + 0.0045$</td>
<td>56.98%</td>
<td></td>
</tr>
<tr>
<td>2008</td>
<td>$y = -0.4623x + 0.0195$</td>
<td>61.36%</td>
<td>$y = -0.1246x + 0.0062$</td>
<td>39.67%</td>
<td></td>
</tr>
<tr>
<td>2009</td>
<td>$y = -0.3189x + 0.022$</td>
<td>61.35%</td>
<td>$y = -0.1893x + 0.0117$</td>
<td>72.41%</td>
<td></td>
</tr>
<tr>
<td>2010</td>
<td>$y = 0.0332x + 0.0117$</td>
<td>16.82%</td>
<td>$y = -0.0163x + 0.0073$</td>
<td>4.94%</td>
<td></td>
</tr>
<tr>
<td>2011</td>
<td>$y = -0.1776x + 0.0109$</td>
<td>34.64%</td>
<td>$y = -0.2068x + 0.0079$</td>
<td>70.33%</td>
<td></td>
</tr>
<tr>
<td>2012</td>
<td>$y = -0.1744x + 0.0121$</td>
<td>20.86%</td>
<td>$y = 0.051x + 0.0132$</td>
<td>3.32%</td>
<td></td>
</tr>
<tr>
<td>2007-2012</td>
<td>$y = -0.2005x + 0.0154$</td>
<td>28.25%</td>
<td>$y = -0.1149x + 0.0084$</td>
<td>43.57%</td>
<td></td>
</tr>
</tbody>
</table>

### Greece

<table>
<thead>
<tr>
<th>Year</th>
<th>Equation</th>
<th>$R^2$</th>
<th>Italy</th>
<th>Equation</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2003</td>
<td>$y = -0.165x + 0.0246$</td>
<td>85.09%</td>
<td>$y = 0.002x + 0.0008$</td>
<td>6.32%</td>
<td></td>
</tr>
<tr>
<td>2004</td>
<td>$y = -0.0195x + 0.0212$</td>
<td>1.18%</td>
<td>$y = -0.0065x + 0.0008$</td>
<td>16.91%</td>
<td></td>
</tr>
<tr>
<td>2005</td>
<td>$y = -0.1657x + 0.0161$</td>
<td>50.12%</td>
<td>$y = -0.0067x + 0.0011$</td>
<td>17.69%</td>
<td></td>
</tr>
<tr>
<td>2006</td>
<td>$y = -0.0007x + 0.0057$</td>
<td>0.03%</td>
<td>$y = -0.015x + 0.0009$</td>
<td>51.59%</td>
<td></td>
</tr>
<tr>
<td>2007</td>
<td>$y = -0.0401x + 0.0069$</td>
<td>4.08%</td>
<td>$y = -0.3597x - 0.0061$</td>
<td>27.09%</td>
<td></td>
</tr>
<tr>
<td>2008</td>
<td>$y = -1.2295x - 0.0105$</td>
<td>57.73%</td>
<td>$y = -0.1647x + 0.0082$</td>
<td>61.86%</td>
<td></td>
</tr>
<tr>
<td>2009</td>
<td>$y = -0.6737x + 0.0246$</td>
<td>69.94%</td>
<td>$y = -0.2273x + 0.017$</td>
<td>61.31%</td>
<td></td>
</tr>
<tr>
<td>2010</td>
<td>$y = 0.0047x + 0.0163$</td>
<td>0.16%</td>
<td>$y = -1.012x + 0.02$</td>
<td>76.42%</td>
<td></td>
</tr>
<tr>
<td>2011</td>
<td>$y = -0.3064x + 0.0082$</td>
<td>77.00%</td>
<td>$y = -0.5439x + 0.0347$</td>
<td>44.51%</td>
<td></td>
</tr>
<tr>
<td>2012</td>
<td>$y = -0.1451x + 0.015$</td>
<td>78.12%</td>
<td>$y = -0.3296x + 0.0121$</td>
<td>20.84%</td>
<td></td>
</tr>
<tr>
<td>2003-2012</td>
<td>$y = -0.2358x + 0.0179$</td>
<td>29.89%</td>
<td>$y = -0.3296x + 0.0121$</td>
<td>20.84%</td>
<td></td>
</tr>
</tbody>
</table>

### Russia

<table>
<thead>
<tr>
<th>Year</th>
<th>Equation</th>
<th>$R^2$</th>
<th>US</th>
<th>Equation</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2007</td>
<td>$y = 0.1309x + 0.00422$</td>
<td>43%</td>
<td>$y = -0.0972x + 0.0038$</td>
<td>47.52%</td>
<td></td>
</tr>
<tr>
<td>2008</td>
<td>$y = -1.2061x + 0.0229$</td>
<td>66.63%</td>
<td>$y = -0.0016x + 0.0044$</td>
<td>0.18%</td>
<td></td>
</tr>
<tr>
<td>2009</td>
<td>$y = -2.4768x - 0.0157$</td>
<td>95.53%</td>
<td>$y = -0.0005x + 0.0051$</td>
<td>0.02%</td>
<td></td>
</tr>
<tr>
<td>2010</td>
<td>$y = -0.0361x + 0.017$</td>
<td>9.78%</td>
<td>$y = -0.0221x + 0.0044$</td>
<td>11.77%</td>
<td></td>
</tr>
<tr>
<td>2011</td>
<td>$y = -0.2783x + 0.0205$</td>
<td>73.18%</td>
<td>$y = -0.0304x + 0.0049$</td>
<td>15.05%</td>
<td></td>
</tr>
<tr>
<td>2012</td>
<td>$y = -0.4554x + 0.0134$</td>
<td>71.18%</td>
<td>$y = -0.0304x + 0.0049$</td>
<td>15.05%</td>
<td></td>
</tr>
<tr>
<td>2007-2012</td>
<td>$y = -0.5532x + 0.0221$</td>
<td>44.08%</td>
<td>$y = -0.0304x + 0.0049$</td>
<td>15.05%</td>
<td></td>
</tr>
</tbody>
</table>
6. Conclusions

Friewald et al. [7] show that equity risk premium and credit default swaps (CDS) spreads are strongly related. Their analysis starts from the Merton [14] structural model: in this framework they show that an inverse relationship between CDS spreads and equity premium holds for a specific firm.

The idea here is that this kind of relation holds also for countries. Considering a simple market with two asset classes only, that is sovereign bonds and domestic equities, if country risk rises, domestic investors should sell sovereign bonds and invest on stocks and vice versa. As a consequence, with regard to premia, this means that increasing sovereign risk requires higher insurance cost, that is higher CDS spreads: investors aiming to reduce their risks or insurance costs should shift to equity markets and this implies equity premium reduction. Again, in other words, sovereign CDS spreads represent the insurance cost against country’s default for bondholders, that is spreads may be intended as a negative risk premium; on the other hand, equity market investors require a positive equity risk premium. All these considerations imply that an inverse relationship between CDS spreads and equity risk premium should hold for countries, as for firms in the theoretical structural framework due to Merton [14].

The main purpose of my work is to investigate empirically if the theoretical ideas above described can be confirmed observing the historical realized premia. With this aim, I have measured sovereign risk directly in terms of CDS spreads on government bonds, while country equity
risk in terms of a suitable reward to risk ratio, that is the excess return of equity market indices over their Conditional Value at Risk (CVaR).

In my paper there is the analysis of the joint behavior of sovereign CDS spreads and of risk premium on countries’ equity indices.

More in detail, I have considered the daily CDS spreads on the 5 years government bonds of a set of countries (Brazil, China, Greece, Italy, Russia and US). Their year by year behavior have been compared with that of the countries daily equity premium defined in terms of Conditional Value at Risk (CVaR) over the period 2002-2012 for Greece, 2003-2012 for Italy, 2008-2012 for US and 2006-2012 for the other countries.

The results of the linear regression of country equity premium over CDS spreads differ from one country to another, but in some years they confirm a very strong inverse relationship between credit spreads and risk equity premium. The negative linear relationship is almost perfect in some of the considered years, e.g. the linear regression equation for Russia in 2009 has a coefficient of determination of 95.53% and that for Greece in 2003 displays a value of 85.09%. In many years and for many countries the R2 index has values greater than 50%. Indeed, there are also many years in which the index has very low values, but I strongly believe that further analysis with more sophisticated statistic instruments should confirm the idea that equity markets and sovereign bonds are strictly linked and that their premia are inversely related: obviously, many factors influence these two markets and some of the simplifications here adopted should be adjusted. These are the guidelines for further developments of the present work.

Last, one of the main implications of this analysis is that equity and credit risk cannot be considered separately as they seem to be strongly negatively correlated: this rules out the possibility of adding together market and credit risk values in order to estimate a global risk measure. This kind of global risk measure should take into account of an important diversification effect.

7. Appendix A

Black and Scholes [5] assume that the given short-term interest rate r is constant through time and that it is possible to borrow and lend money at the rate r, that the underlying asset does not pay dividends, that short selling is possible, that there are no transaction costs with continuous trading and that financial markets are arbitrage free. Moreover, they assume that the underlying price, here V(t), follows a geometric brownian motion under the (real world) measure P, that is:

\[ dV(t) = V(t) \cdot (\mu dt + \sigma dW^P(t)) \]  

Consider now the price X(t ;V(t)) of a financial instrument depending on t and V(t). Using Ito’s lemma (e.g. see Bingham, Kiesel [3]) it is possible to show that X satisfies the following stochastic differential equation:

\[ dX(t, V(t)) = X(t, V(t)) \cdot (\mu^P dt + \sigma^P dW(t)) \]

where the parameters of the process \( \mu^P \) and \( \sigma^P \) are, respectively, the mean and standard deviation of \( dX / X \) (that is the local return of \( X(t, V(t)) \)) under the risk measure \( P \). It can be shown by Ito’s lemma that:
\[\mu_X = X_t + \mu X_V + \frac{1}{2} \sigma^2 X_{VV} = E^p \left( \frac{dX}{X} \right); \quad \sigma_X^p = \sigma X_V \quad (7)\]

where the subscripts denote partial derivatives.

Under the risk neutral measure \(Q\), the price \(V(t)\) satisfies the following process instead of that ruled by equation 6:

\[dV(t) = V(t) \cdot (r dt + \sigma dW^Q(t)) \quad (8)\]

and the price \(X(t; V(t))\):

\[dX(t, V(t)) = X(t, V(t)) \cdot (\mu^Q dt + \sigma^Q dW(t))\]

Applying again Ito’s lemma, the risk neutral drift and volatility of the return \(dX/X\) under the neutral measure \(Q\) are given by:

\[\mu^Q_X = X_t + r X_V + \frac{1}{2} \sigma^2 X_{VV} = E^Q \left( \frac{dX}{X} \right); \quad \sigma^Q_X = \sigma X_V \quad (9)\]

Dening the market price of risk of \(X(t, V(t))\) as:

\[\lambda_X = \frac{\mu_X^p - r}{\sigma_X^p} \]

that is:

\[\lambda_X = \frac{E^p \left( \frac{dX}{X} \right) - E^Q \left( \frac{dX}{X} \right)}{\sigma_X^p} \]

or, again:

\[\lambda_X = \frac{\mu - r}{\sigma} \frac{X_V}{|X_V|} \quad (10)\]

Both the call and the put option are instruments on the firm’s value \(V(t)\) and the above considerations hold for them.

The call and the put option Black and Scholes prices are:

\[c = V(t) N(d_1) - D \cdot e^{-r(T-t)} N(d_2)\]
\[p = D \cdot e^{-r(T-t)} N(-d_2) - V(t) N(-d_1)\]

with \(N(\cdot)\) the cumulative normal distribution function and \(d_1\) and \(d_2\) given by the following conditions:

\[d_1 = \frac{\ln(V(t)/D) + (r + \sigma^2/2)(T-t)}{\sigma \sqrt{T-t}}\]
\[d_2 = d_1 - \sigma \sqrt{T-t}\]

The partial derivatives with respect to the underlying (the so called delta) involved in equations 10 are:
\[ c_V = N(d_1); \quad p_V = -N(-d_1) \]

Hence, \( c_V \) has values in the interval \((0, 1)\) while \( p_V \) in \((-1, 0)\) and this implies:

\[ \lambda_c = \frac{\mu - r}{\sigma} = -\lambda_p \]

References