

## Skewness-Based Portfolio Selection: Implications for International Investing in Frontier Markets

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**Abstract** Using the Morgan Stanley Capital International index monthly data for 28 frontier markets during the period between January 2011 and December 2018 the paper investigates the impact of skewness on portfolio selection. The existence of skewness in returns distributions is illustrated in terms of frontier markets. The skewness ranks matrix is developed. It demonstrates that only 6 markets can be regarded as not skewed from USD and 11 – from EUR perspective. The study does not find strong evidence on positive or negative skewness character. All in all, skewness for local currencies is slightly higher than for foreign currencies. It is the factor of international portfolio investing in frontier markets but its impact should not be overestimated. Analytical framework for skewness-based investing in frontier markets is developed. It does not indicate strong evidence that skewness is a more important portfolio selection factor for international investments than for domestic ones. Skewness is rather more relevant for domestic portfolio investing. Using the approach of relative foreign exchange percentage differential, the study proves the more notable impact of skewness for EUR than for USD international investors. As to the preferable moment, the found evidence is weak but rather in favor of skewness than return for local investing and in favor of return for international investing.

**Keywords:** international portfolio investing; skewness; frontier markets; returns distribution moments; relative foreign exchange percentage differential

**JEL Classification:** F21; G11; G15; C46

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## 1. Introduction

Skewness and kurtosis of returns distributions is something known enough in the field of international investing while being something rather recent at the same time. On the one hand the idea of downside risk consideration originates from early 1950-s and its development began together with the portfolio theory itself. On the other hand, the contemporary build-up of post-modern portfolio theory can be dated not earlier than mid 1990-s and this theory is intensively developing nowadays. The principal ideas of post-modern portfolio theory can be summarized as follows. First, considering not only traditional risks and returns while investing in portfolios of securities but skewness of returns distributions as well, and later – kurtosis of such distributions. In mathematical term it goes about the third and the fourth moments of random variable distribution. Second, considering downside risk that is often much more relevant risk measure than traditional variance or standard deviation. In fact, the pure version of post-modern portfolio theory implies just the third moment of returns distribution – skewness. As to the kurtosis that is the fourth moment of random variables returns distribution, it also refers to the post-modern portfolio theory though this idea appeared later.

Many empirical studies support the concept of post-modern portfolio theory confirming that using higher moments of returns distributions in portfolio strategies often results in much more efficient portfolios. In these terms, we should stress that these higher moments do not change the existing paradigm of portfolio selection. Markowitz portfolio theory still represents the underlying fundamental approach while the post-modern portfolio theory is actually just introducing additional portfolio selection parameters such as skewness and kurtosis. Theoretical considerations give the grounds to talk about certain internal shift of the paradigm and its improvement rather than about the changes of the paradigm itself. Allowing for above mentioned, no wonder that the following definite questions naturally appear. Is the new higher moments approach really more effective for portfolio investors? If so, what is the scope of this difference, and does this scope stand for domestic or international portfolio investing?

Answering the pointed questions in terms of international investing requires considering exchange rate exposure, country risks, market risks of different countries and other issues that are not faced by portfolio investors in local markets. However, even within the international portfolio investing universe there are strongly defined clusters of countries identified using different criteria. These criteria can be completely different though the typical ones can include geography, type of returns distribution, level of market development etc. The last one is used most often. It implies that equity markets are typically divided into three groups: developed, emerging and frontier markets. In our study we focus on the least developed group – frontier markets. These markets are traditionally identified as riskier considering general and exchange rate risk, less reliable in terms of country and political risks, less developed in the light of institutional environment and market infrastructure. These markets are the main focus of the present study.

## 2. Literature review

The issue of application of higher moments of returns distribution in portfolio selection has been widely studied in the literature. Notwithstanding that early portfolio theory utilized just the simple standard deviation as a risk measure, Markowitz (1959) suggested to use the semivariance for estimating portfolio and individual securities risks. Unlike the variance, the semivariance considers just the values lower than the average. Such approach focuses on a so-called downside risk. This idea became fundamental but not sufficient to complete the composition of post-modern portfolio theory. The last one is agreed to become formally matured in mid-1990-s and refers to the study of Rom & Ferguson (1993). They actually introduced the term 'post-modern portfolio theory' and identified the theory itself as the more general version with the modern portfolio theory being the particular detached version for the case of symmetric returns distribution. This point is the second fundamental and at the same time sufficient to finalize the methodology of skewness-based investing. However, the downside risk utilization was going on during the whole prior period.

The principal precondition to expand the traditional methodology of portfolio selection using the third moment is that in practice most returns are actually distributed asymmetrically, undermining the basic assumption of traditional portfolio theory about normal distribution of returns. Considering the downside risk idea investors would rather prefer the right-tailed distributions, since the upside risk is not really the risk in terms of possible losses. This setting also corresponds with basic behavioral condition according to which investors would rather not suffer from losses than get profits. In the empirical part of their study Rom & Ferguson (1993) compared three portfolios: minimum variance portfolio, maximum efficiency portfolio and equivalent risk portfolio from American investor's perspective. They generally conclude that portfolios composed using the downside risk (relative to minimum accepted return – *MAR*) are more appropriate and allow for much higher level of diversification. Outstanding developments in the field of skewness analysis in portfolio management cover the elaboration of the new risk adjusted return ratio – Sortino ratio. As well as the traditional Sharpe ratio the new index developed by Sortino & Van Der Meer (1991), Sortino & Price (1994) and Sortino (2001) implies the relation of a risk premium to standard deviation. However, the risk premium itself is computed in another way – as a difference between expected return and *MAR* but not the risk free rate. The standard deviation also differs and not just technically. It is a target downside deviation calculated using the new risk premium values. Many contemporary studies confirm the reasonability of using Sortino ratio for portfolio optimization such as Dey & Mitra (2012), Galloppo (2010), Rollinger & Hoffman (2013), Washer & Johnson (2013), Swisher & Kasten (2005) etc. However, Washer & Johnson (2013) point out that though Sortino ratio produce mostly better results major researchers and practitioners go on using the traditional Sharpe ratio as well.

Among the first to prove the skewness validity in portfolio selection was Arditti (1967) who showed that unlike the correlation between asset and market returns the second and third moments of returns distribution are relevant risk measures while

the geometric mean is considered as the average or an expected return. The author states that the first three moments contain all the information about assets' returns. The positive (right-tailed) distributions are proved to be preferred by investors. Samuelson (1970) emphasized the traditional mean-variance approach for the case of a so called 'compact' probabilities when risk levels are low. Though the traditional view is generally supported by the author he suggests the more general  $r$ -moment model while skewness being more appropriate for the above mentioned case. Lai (1991) proved that considering skewness brought an investor to optimize a portfolio with contradicting objectives, such as maximizing return and skewness while minimizing risk at the same time was impossible. Thus, investor's final decision depends on the preference between objectives, and the optimal portfolio always exists. This is also true for kurtosis and higher moments. However, in a three-dimensional framework the traditional Markowitz efficient portfolio can become inefficient while the risk-return-skewness portfolio can be inefficient in traditional understanding. Sun & Yan (2003) state that positive ex post skewness is persistent for individual securities but not for portfolios, that weakens its positions in portfolio selection. The reason is that such portfolios are efficient only in terms of mean and variance. They instead test the persistency of mean-variance-skewness efficient portfolios for Japanese and American investors and find out that considering the skewness preference enhances portfolios' persistency over time.

Mencía & Sentana (2009) focus on more technical issues of portfolio selection considering skewness. They analytically derive the mean-variance-skewness frontier formula under assumption of location-scale mixture of normal distributions of portfolio components. The efficient three moments frontier implies the maximum skewness for any given combination of mean and variance and is obviously built up in a three dimensional system of coordinates. The study also shows that new efficient set can be spanned by three funds: two traditional funds (risk-free and risky assets) for mean-variance set and a skewness-variance efficient set (maximum skewness for a given variance). Kerstens, Mounir & Van de Woestyne (2011) specify crucial aspects of mean-variance-skewness efficient frontier geometric presentation that is very suitable for portfolio selection but brings an investor again to the utility function that is now much more complicated. In three-dimensional framework investors have some skewness preferences and are ready for an increased risk thereof. Utilizing the shortage function as a measure of efficiency the study supports the general idea of dominating portfolio paradigm that efficient portfolios are not diversified enough while diversified portfolios are not efficient. This is true for mean-variance as well as for mean-variance-skewness portfolios. The three dimensional efficiency is always smaller than the two-dimensional one. Further Bricc, Kerstens & Van de Woestyne (2013) expand the mean-variance-skewness analysis comparing the more traditional polynomial-goal programming (PGP) approach and the shortage function approach finding the point where they can be matched under risk-free asset existence. They introduce a so-called unit variance mean-skewness section being generated using the shortage function with a fixed level of variance that equals unity. PGP portfolios are located on this section that is in turn part of a more general mean-variance-skewness set and they are efficient in three dimensions.

Conrad, Dittmar & Ghysels (2013) explore the relation between volatility and skewness on the one hand and returns on the other. Using the option data, they find out that higher moments do impact the further returns. Particularly high volatility brings about low returns, and vice versa. High skewness in turn results in lower subsequent returns and the general relation is negative confirming investors' preference towards positive skewness. The study also proves that implied distributions are much more stable than historical ones. For implied physical distributions there exists a reverse relation between skewness and expected traditional Sharpe ratio. Similar option prices based approach (in terms of implied volatility) was utilized by DeMiguel et al. (2013). The authors argue that using option-implied information can improve portfolio performance; particularly exploiting option-implied volatility reduces the portfolio volatility. Option-implied skewness brings about a substantial increase in portfolio's Sharpe ratio either with short selling and transaction costs or without them. Bhattacharyya, Hossain & Kar (2014) develop the fuzzy portfolio optimization model that operates a multi-objective algorithm implying maximization of expected return and skewness while minimizing portfolio variance and cross-entropy. The last ratio shows the accuracy of probabilistic forecasts in terms of deviation of returns from its desired value. Jiang, Ma, An (2016) consider systematic skewness in portfolio selection by which they mean the normalized asset's co-skewness reflecting the co-movement between asset's return and market squared return. It is proved that the necessary coskewness can be achieved at the expense of traditional efficiency. Efficient portfolios composed using this mean-variance-coskewness approach have higher skewness than traditional mean-variance efficient portfolios.

Some not so clear evidence regarding the interrelation between individual assets and portfolio skewness can be found in Kim (2015). It is empirically shown that variables which are traditionally considered to define the portfolio skewness (coskewness uppermost) do not do this in practice. Few theories can bind positive coskewness and negative portfolio skewness, while the number of securities in a portfolio shows much stronger impact. Kim et al. (2018) investigate the portfolio three-dimensional risk including not only the traditional standard deviation but skewness and kurtosis as well. They find out that all others equal increasing the number of securities in a skewed fat-tailed portfolio brings about the decrease of variance as well as skewness and kurtosis. Herewith skewness diminishes much slower than variance and kurtosis meaning weaker possibilities to diversify it.

More specific in terms of our study country selection is the research by Cenela & Collazo (2007) that focuses on different industries in emerging markets. The polynomial goal programming approach utilized by the authors showed that considering skewness altered the optimal portfolios' structure substantially and required investors to equilibrate between expected return and skewness. Still the study confirms the asymmetry of distributions for 46 industrial indices in emerging markets that makes asymmetry an appropriate international portfolio selection factor. Todonti (2015) reveals two shortcomings of traditional for post-modern portfolio theory Sortino ratio. It does not consider the two sub-parts of the returns distribution: unrealized returns (positive

but lower than the average, or the mode, or the target) and losses (negative returns). The author instead suggests the alternative risk-adjusted measure that utilizes a special global risk rather than the target return. It is computed using the multipliers method. The suggested global risk measure is actually not a risk in terms of geography but in terms of the whole returns distribution range coverage, including losses and unrealized returns. The developed multipliers method implies that for all of the above mentioned sub-parts returns are explored with different weights defined by special multipliers. Using the elaborated approach along with the traditional Sortino ratio, the paper investigated the five Central and Eastern European markets concluding that skewness does matter in international portfolio investing while the proposed measure being more relevant than the traditional one.

### **3. Hypothesis, methodology and data**

Although there exists a deep pile of scientific research devoted to different aspects of considering skewness in international portfolio investment decisions, only few studies investigate frontier markets. That can be quite distinct allowing for the low level of these markets development and extremely little share of their international portfolio assets and liabilities. In most cases such markets are unstable, have highly volatile returns and exchange rates but are still potentially attractive for foreign investors. In our research we are going to explore the level of frontier equity markets skewness and infer these results into the space of international portfolio investing strategies formulation. Herein we put forward two working hypotheses. First, frontier markets return distributions are skewed and this skewness is positive making the post-modern portfolio theory framework appropriate for these markets in terms of foreign investors holdings of these markets assets in their portfolios. This hypothesis appeals since most empirical research as well as theoretical and methodological findings concern developed markets and in few cases emerging markets. Frontier markets are poorly explored though they represent extremely high potential of international portfolio investing. Second, skewness is different when measured in different currencies, particularly different foreign currencies and local currency. Contemporary portfolio investing theory is unique in terms of its being universal. It is valid for local investing as well as for international. There is no special paradigm for international investing; it uses the traditional Markowitz approach as well as other theories and concepts. However, of central interest is the question of how appropriate the post-modern portfolio theory is for domestic and for international portfolio investing.

These hypotheses will be tested using the Morgan Stanley Capital International (MSCI) index data (MSCI-1, 2019). The main point of our analysis are the indices for individual frontier markets and the group index as a benchmark. As of April 30, 2019 the frontier markets group covered the following markets (MSCI-2, 2019): Argentina, Jamaica, Panama, Trinidad & Tobago (Americas sub-group); Croatia, Estonia, Lithuania, Kazakhstan, Romania, Serbia, Slovenia, Bosnia & Herzegovina, Bulgaria, Ukraine

(Europe & CIS sub-group); Kenya, Mauritius, Morocco, Nigeria, Tunisia, WAEMU<sup>1</sup>, Botswana, Ghana, Zimbabwe (Africa sub-group); Bahrain, Jordan, Kuwait, Lebanon, Oman and Palestine (Middle East sub-group); Bangladesh, Sri Lanka and Vietnam (Asia sub-group). Out of these 34 markets the following 10 are not presented in the group index: Jamaica, Panama, Trinidad & Tobago, Bosnia & Herzegovina, Bulgaria, Ukraine, Botswana, Ghana, Zimbabwe, Palestine. However, MSCI do not provide the index data for Palestine and the named WAEMU markets, so our selection will cover 30 markets except the mentioned ones.

As to the period in question, it had better be as long as possible but here the technical problem of data availability arises. The longer the period the more markets must be excluded from the research. In these terms, we decide on the following compromise: exclusion of only one market (Panama) allows us starting the period since January 2011. Besides the mentioned technical issue, we consider two more reasons here. First, Panama is not the market that attracts portfolio investors by its real investment characteristics, including returns, variance and skewness. It is a known offshore center where regulation is the major factor pulling foreign capital. Second, the financial and statistical vision of the period selected tells us that we have 8 years that make 96 months (periods) in total. All others equal 60 periods are considered to be statistically appropriate. However, we still exclude Ghana for which the data for 2018 is not provided. That finally makes our dataset consisting of 28 markets and a benchmark frontier markets index (FM).

Using the respective index values and the standard methodology we compute indices' monthly returns and their standard deviations. Though there are several approaches to skewness computation (see Brys, Hubert & Struyf (2004) for example) we use the most traditional one implying the relation of the third moment to the second moment raised to the 2/3<sup>rd</sup> power:

$$R_{SKEW} = \frac{\sum_{i=1}^n w_i (r_i - \bar{r}_i)^3}{\sigma(r_i)^3}, \quad (1)$$

All indices' values cover the standard capitalization range including large, medium and small-capitalized firms. The index level considers prices only. Distribution moments statistics is calculated using the index values in USD, EUR and local currencies.

The relative foreign exchange percentage differential will be used to compare the relative importance of return and skewness in foreign currencies as to the local currency. This methodology was described and used by Rogach & Dziuba (2017) and implies the computation of a relation of difference between returns (skewnesses) in local and foreign currencies to the return (skewness) in local currency multiplied by hundred percent. Moreover, since the most important issue of this methodology is to retain the sign from the numerator (it demonstrates the unknown difference) we take the divisor in magnitude. Thus, the results should be interpreted as follows: negative values of the

<sup>1</sup> West African Economic and Monetary Union (WAEMU) includes the markets of Benin, Burkina Faso, Ivory Coats, Guinea-Bissau, Mali, Niger, Senegal and Togo with securities from only Senegal, Ivory Coast and Burkina Faso markets being considered for some sub-regional indices. Individual indices for these markets are not provided.

differential testify the relative attractiveness of investing in a foreign currency while the differential positive sign means that local currency investing is more relevant all others equal.

## 4. Results

### 4.1. Estimation of markets skewness

Computation results are presented in Table 1.

**Table 1.** Three moments of frontier markets returns distribution (January 2011 – December 2018)

Market	$\bar{r}_i$			$\sigma(r_i)$			$R_{SKEW}$		
	USD	EUR	Local	USD	EUR	Local	USD	EUR	Local
Argentina	0.09	0.21	0.09	11.46	11.16	11.46	0.25	0.42	0.25
Bahrain	-1.19	-1.03	-1.19	5.15	5.13	5.12	-0.37	0.01	-0.38
Bangladesh	-0.24	-0.05	-0.08	6.85	7.29	6.63	-0.74	-0.68	-0.78
Bosnia and Herzegovina	-0.82	-0.69	-0.69	5.15	4.34	4.34	0.71	0.78	0.78
Botswana	-0.73	-0.58	-0.26	6.08	5.86	5.57	-0.31	-0.28	0.06
Bulgaria	-0.25	-0.16	-0.16	7.65	6.67	6.66	1.17	1.46	1.46
Croatia	-0.37	-0.24	-0.24	4.54	3.48	3.39	0.39	0.37	0.40
Estonia	0.20	0.32	0.32	6.22	5.35	5.35	0.27	0.62	0.62
Jamaica	1.31	1.48	1.72	5.99	6.11	5.69	0.79	0.65	0.69
Jordan	-0.44	-0.26	-0.44	4.62	4.97	4.60	0.85	0.94	0.85
Kazakhstan	-0.21	-0.07	-0.21	7.49	7.28	7.49	-0.10	0.10	-0.10
Kenya	0.62	0.77	0.83	6.04	5.83	5.56	-0.31	-0.34	-0.15
Kuwait	-0.26	-0.10	-0.18	4.16	4.05	4.05	0.17	0.08	0.16
Lebanon	-0.54	-0.35	-0.54	3.82	4.44	3.82	0.94	1.11	0.94
Lithuania	-0.24	-0.12	-0.12	4.51	3.37	3.37	-0.14	-0.11	-0.11
Mauritius	-0.07	0.10	0.03	3.69	3.62	3.35	-0.38	-0.69	-0.55
Morocco	-0.40	-0.25	-0.27	4.22	3.77	3.77	0.35	0.42	0.46
Nigeria	-0.33	-0.18	0.49	7.41	7.18	6.37	-0.43	-0.37	0.10
Oman	-0.54	-0.36	-0.54	4.28	4.65	4.28	-0.37	0.00	-0.38
Romania	0.62	0.70	0.78	7.06	5.74	5.44	-0.60	-0.81	-0.85
Serbia	-0.06	0.02	0.11	7.19	6.07	5.69	-0.33	0.01	0.33



Market	$\bar{r}_i$			$\sigma(r_i)$			$R_{SKEW}$		
	USD	EUR	Local	USD	EUR	Local	USD	EUR	Local
Slovenia	-0.17	-0.07	-0.07	5.66	4.22	4.22	-0.08	0.06	0.06
Sri Lanka	-0.46	-0.28	0.04	5.70	5.83	5.34	0.17	0.04	0.20
Trinidad and Tobago	0.73	0.93	0.79	3.16	3.94	3.15	0.33	0.18	0.30
Tunisia	-0.23	-0.07	0.52	4.57	4.47	4.13	0.47	0.22	0.46
Ukraine	-1.68	-1.55	-0.66	9.92	9.68	9.74	-0.36	-0.12	0.49
Vietnam	0.09	0.23	0.25	6.39	5.95	6.14	0.18	0.11	0.30
Zimbabwe	2.89	3.08	2.89	14.95	15.19	14.95	2.18	2.19	2.18
FM	-0.10	0.05	0.10	3.37	2.98	3.12	-0.33	-0.34	-0.36
Average	-0.10	0.05	0.11	6.21	5.92	5.70	0.17	0.23	0.28

Note: calculated by the authors using MSCI-I (2019) data.

In order to address the computation results we need some further analytical developments. Particularly we need to fix some relative anchors for skewness ranges, since the figures themselves are difficult to be explained. The ranges fluctuate between -0,74 and 2.18 for USD, between -0.81 and 2.19 for EUR and between -0.85 and 2.18 for local currencies. Though the last range has weak financial contents since the range covers skewnesses in different currencies and can be analyzed only technically, the first two ranges are equal and comparable. Therefore, we take that every range is a 100 % – the whole band of observed skewness values. The respective ranges in absolute figures are 2.91 points for USD, 3.00 points for EUR and 3.03 points for local currencies. As we see the ranges do not differ substantially in absolute values, with the local currencies skewness being slightly higher than for USD and EUR. However, dealing with the first hypothesis we need to decide upon the skewed and unskewed (normal) distributions. Statistically the normal distribution is observed when skewness is zero. It is empirically a very rare case since distributions can be very close to zero being statistically skewed but actually normal considering investment decisions.

Let's consider 10 % from zero for each tail to be the skewness absence. Taking the low skewness being in the 30 % range from zero for each tail, the medium – in the 50 %, the strong in the 75 % and the very strong in the 90 % from zero for each take we finally get five ranks of skewness. Since the right tails of the observed distributions are much heavier than the left ones we take the mentioned percentages as of the right tail absolute length. Absolute skewness values for each rank and respective markets are presented in Table 2.

**Table 2.** Skewness ranks, values and markets ranking

Rank	Skewness Ranks	Absolute Values	USD	EUR
1	Very strong (+)	1.962 – 2.19	Zimbabwe	Zimbabwe
2	Strong (+)	1.635 – 1.962	–	–
3	Medium (+)	1.090 – 1.635	Bulgaria	Bulgaria, Lebanon
4	Low (+)	0.654 – 1.090	Lebanon, Jordan, Jamaica, Bosnia and Herzegovina	Jordan Bosnia and Herzegovina
5	Very low (+)	0.218 – 0.654	Tunisia, Croatia, Morocco, Trinidad and Tobago, Estonia, Argentina	Jamaica, Estonia, Argentina, Morocco, Croatia, Tunisia, <b>Average</b>
6	Normal and close to normal	-0.218 – 0.218	Vietnam, Kuwait, Sri Lanka, Slovenia, Kazakstan, Lithuania, <b>Average</b>	Trinidad and Tobago, Vietnam, Kazakhstan, Kuwait, Slovenia, Sri Lanka, Bahrain, Serbia, Oman, Lithuania, Ukraine
7	Very low (-)	-0.654 – -0.218	Botswana, Kenya, <b>FM</b> , Serbia, Ukraine, Bahrain, Oman, Mauritius, Nigeria, Romania, Bangladesh	Botswana, <b>FM</b> , Kenya, Nigeria
8	Low (-)	-1.090 – -0.654	–	Bangladesh, Mauritius, Romania
9	Medium (-)	-1.635 – -1.090	–	–
10	Strong (-)	-1.962 – -1.635	–	–
11	Very strong (-)	-2.19 – -1.962	–	–

Notes:

1. Calculated and composed by the authors using MSCI-1 (2019) data.

2. Markets with normal or close to normal distributions as well as respective ranks and absolute values are marked with grey filling.

Our findings confirm the general idea about returns skewness. Most frontier markets returns are skewed for international investors either in terms of USD or EUR. Considering the above defined ranks we conclude that from USD perspective just 6 out of 28 markets have normal or close to normal distributions of returns. Other markets demonstrate skewness with 15 being positively skewed and other 13 – negatively. Most markets have low or very low skewness with two of them standing out: one market has medium positive skewness, and the other has very strong positive one. From EUR investor perspective we observe the similar implications. The main difference is that instead of 6 markets normal

or close to normal distributions can be observed for 11 out of 28 explored markets. This means that skewness-based international investment strategies in frontier markets are more relevant for USD than for EUR investors. Positive skewness is common for 20 markets, while 8 out of 28 markets have negative skewness. Another distinction from the USD skewness is that three markets have low negative skewness.

Special attention should be paid to skewness of frontier markets group as a whole (FM) and the average value (Average). These values differ with the first being negative and the second positive. Our basic argument is that Average ratio is more relevant in terms of potential investing. So actually, we can consider the frontier markets to have positive skewness rather than negative one. The minus sign for FM results from some highly capitalized markets with negative skewness since this ratio is a weighted average of its components. So one relatively large market with negative skewness can bring about negative skewness of the weighted ratio overbalancing several relatively small markets with positive skewness. Nevertheless, an investment strategy implies that an investor considers specific markets meaning their precise investment characteristics rather than a statistical metric that is an FM index. Small markets can have lower impact on an index but attract foreign investors as well as large markets. Thus, the group average ratio is of a higher relevance for the purposes of our research. All in all, frontier markets have very low positive skewness or are close to normal distributions.

#### 4.2. Skewness exchange rate impact and preferable distribution moment

So far, we investigated skewness in terms of foreign currency only and did not deal with local currencies. Nevertheless, an important issue is to compare skewnesses in local currency and in a foreign currency, USD and EUR in our case. Such analysis would allow us to test the second hypothesis on relative importance of skewness in domestic and international investing. Furthermore, we will also estimate the relative relevancy of the first and third moments of returns distribution. They both are to be maximized and can be mutually substituted as shown by Cenela & Collazo (2007). We shall carry out such analysis using the relative foreign exchange percentage differential for returns as well as for skewnesses that are presented in Table 3.

**Table 3.** Relative foreign exchange percentage differential for return and skewness

Market	USD		EUR		Market	USD		EUR	
	$r_i$	$R_{SKEW}$	$r_i$	$R_{SKEW}$		$r_i$	$R_{SKEW}$	$r_i$	$R_{SKEW}$
Argentina	0.00	0.00	-133.33	-65.92	Mauritius	333.33	-31.49	-233.33	24.87
Bahrain	0.00	-3.16	-13.45	-103.55	Morocco	48.15	24.17	-7.41	8.91
Bangladesh	200.00	-6.09	-37.50	-13.80	Nigeria	167.35	513.75	136.73	461.47
Bosnia and Herzegovina	18.84	9.45	0.00	0.04	Oman	0.00	-0.98	-33.33	-100.69
Botswana	180.77	635.19	123.08	596.34	Romania	20.51	-28.86	10.26	-4.76
Bulgaria	56.25	20.30	0.00	0.39	Serbia	154.55	200.61	81.82	96.90

Market	USD		EUR		Market	USD		EUR	
	$r_i$	$R_{SKEW}$	$r_i$	$R_{SKEW}$		$r_i$	$R_{SKEW}$	$r_i$	$R_{SKEW}$
Croatia	54.17	1.49	0.00	6.56	Slovenia	142.86	251.53	0.00	0.00
Estonia	37.50	56.10	0.00	0.00	Sri Lanka	1250.00	16.21	800.00	79.87
Jamaica	23.84	-13.99	13.95	5.86	Trinidad and Tobago	7.59	-8.50	-17.72	40.93
Jordan	0.00	0.11	-40.91	-9.53	Tunisia	144.23	-3.47	113.46	52.22
Kazakhstan	0.00	-0.21	-66.67	-206.07	Ukraine	154.55	174.38	134.85	125.58
Kenya	25.30	109.66	7.23	125.58	Vietnam	64.00	38.99	8.00	63.33
Kuwait	44.44	-9.63	-44.44	51.16	Zimbabwe	0.00	0.00	-6.57	-0.73
Lebanon	0.00	0.02	-35.19	-18.27	FM	200.00	-8.22	50.00	-6.00
Lithuania	100.00	33.92	0.00	0.00	Average	190.91	39.29	54.55	17.86

Note: calculated and composed by the authors using MSCI-1 (2019) data.

Analysis of computed differentials allows for the following conclusions. The skewness factor of portfolio selection in frontier markets is relatively more favorable for international USD investors rather than for domestic ones in case of 10 out of 28 markets. For EUR investors the respective figure is 9 markets though the differential values are much higher for EUR. However, the total quantity of countries should be reduced from 28 to 26 for USD (with Argentina and Zimbabwe being excluded) and to 25 for EUR (with Estonia, Lithuania and Slovenia being excluded) since the mentioned countries have zero returns and skewnesses that can be explained by their monetary policy and pegging of their currencies to world currencies. The group index speaks for international investing while the average – for domestic with no rigorous conclusion. However, in this case we consider the average to be much more relevant figure that can be explained by the following. Group index is a statistically computed figure which performs informational and analytical function mostly. It is calculated usually as a weighted average and thus more capitalized markets account for higher impact on the final result. Consequently, one large market for example can easily overweight several smaller ones. Such ratios are typically used by investors as indicators. Once they are really ready to invest the figure for certain markets they are going to enter becomes much more important to them. This content is actually demonstrated by the average rather than the index.

As to the preferable moment, we acknowledge that for USD investors there is no case where the average and the skewness appear in international investing at the same time. In seven cases the differentials confirm the advantages either of international or domestic investing with them having opposite signs. Only three markets (Bahrain, Kazakhstan and Oman) have zero values of the differential for the average and the negative values for the skewness. No return figure testifies the benefits of international investing, all having positive signs or equating to zero. We also find that for 17 markets

the return differential exceeds the skewness differential meaning that decisions on international investing on this market would be defined rather by the return than by skewness. This is valid either for cases where skewness is negative (seven markets) or positive (seven markets). For nine markets with either positive or nonnegative differentials skewness differential is higher testifying in favor of skewness rather than return while taking investment decisions. However, such analysis should be carried out for cases where differentials have identical signs, which will be done later. We still keep in mind that Zimbabwe and Argentina are excluded from the analysis since they have zero values for both differentials and we have just 26 markets under question. As to the FM group figure, it speaks for international investing in terms of skewness and for domestic investing in terms of return with the return differential being much higher than that of skewness. The average figure that is regarded to be more meaningful in our case, is strongly for domestic investing by both differentials. The summary of these results is generalized in Table 4.

**Table 4:** Analytical framework of international investing in frontier markets considering first and third moments of returns distributions

Relation Type	Quantity of Markets	
	USD	EUR
+ / +	14 & Average	9 & Average
- / -	–	8
+ / -	7 & FM	1 & FM
- / +	–	4
0 / 0	2	3
0 / +	3	3
0 / -	2	0
<b>Total</b>	<b>28</b>	<b>28</b>
Within Identical Sign		
> (+)	7 & Average	4 & Average
< (+)	7	5
> (-)	–	5
< (-)	–	3
<b>Total</b>	<b>14 &amp; Average</b>	<b>17 &amp; Average</b>

Notes:

1. Calculated and composed by the authors using data from Table 3.
2. The relation type column in the first part of the table should be read as follows. The first sign (before slash) is the sign of the return differential. The second sign (after slash) is the sign of the skewness differential.
3. The relation type column in the second part of the table should be read as follows. “>” means that return differential exceeds the skewness differential. “<” means that return differential is lower than the skewness differential. The sign in brackets is the sign of both differentials and it is always identical.

Rather different context applies to international investing in EUR. There are eight markets where we observe both negative differentials that is an argument in favor of international investing. Moreover, four markets with positive skewness differential have negative return differentials. There are nine markets with both positive differentials. Three markets having zero differential values are still excluded from the analysis. Different sign is observed for just five markets. Among markets with two positive differentials five have higher skewness differential and four – higher return differentials. Respective figures for cases with both negative signs are three and five. The average figure is for domestic investing as well.

## 5. Conclusions

Using the approach of relative foreign exchange percentage differential, we have tested the two hypotheses concerning skewness and return in international portfolio investing on frontier markets. Computing skewness and average returns for 28 markets in the period between January 2011 and December 2018 we confirm the point that returns distributions in frontier markets are skewed. The developed skewness ranks matrix allows to conclude that only 6 out of 28 markets under question have normal or close to normal distribution of returns with others having either positive or negative skewness from USD perspective with no strict aptitude to either sign. Most markets have low or very low skewness. From EUR perspective 11 markets are not or hardly skewed in terms of their returns distributions while other 20 are positively skewed, and the remaining 8 markets are negatively skewed. This means the higher impact of skewness for EUR than USD international investors, though local currencies skewness is slightly higher than that of the mentioned currencies. The resulting implication is that skewness emerges to be the factor of international portfolio investing in frontier markets but its impact should not be overestimated.

We developed an analytical framework for skewness-based investing in frontier markets that allows comparing relative attractiveness of domestic and international investing considering different currencies. We have not found strong evidence that skewness is a more important portfolio selection factor for international investments than for domestic. Moreover, our study is rather to confirm that skewness is more important for domestic portfolios. Only 10 out of actual 26 cases speak in favor of USD international investors rather than for domestic ones. Respective figure for EUR is 10 out of 25 markets.

The more substantial impact of skewness for EUR than for USD investors is also justified by the fact that for USD there is no market where the returns and skewness differentials are negative at the same time whereas all return differentials are positive or equal to zero that is in favor of domestic investing. From EUR perspective there are eight markets where both differentials are negative that is an incentive for international investing. Four markets with positive skewness differential have negative return differentials in contrast to the case for USD. Among markets with two positive differentials, five have higher skewness differential and four – higher return differential.

Respective figures for cases with both negative signs are three and five. The average figure is for domestic investing as well.

However, the mentioned results should be treated critically and further research is still required. Its three main directions should be stressed. First, specific optimization problems should be solved, tested and compared. Second, comparison with other market groups would be meaningful. This would allow to find the relative location of frontier markets as to other market groups. Third, risk aversion could be a substantial factor of choosing between return and skewness. By this we mean not the traditional risk tolerance but the risk tolerance towards return and skewness.

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