The Behaviour of US and UK Public Debt
Further Evidence Based on Time Varying Parameters

Suleyman Bolat • Aviral K. Tiwari • Mihai Mutascu

Abstract  The paper investigates whether US and UK have followed sustainable debt policies during the period 1970-2012, by exploring the reaction of the primary surplus as percentage of GDP to variations in the debt to GDP ratio, as a powerful test. The main results reveal that the coefficient for UK is negative and significant, while for the US, we are unable to find a clear-cut evidence of the sustainability of public debt as the coefficient is also negative, but insignificant. In the case of the UK, the outputs reveal that government did not raise the primary surplus as the government debt increased rather reduced it and this reduction has been significant. On the other hand, the significance of the reaction coefficient demonstrates that the reaction of the primary surplus to increases in public debt varies over time. All these evidences allow us to appreciate that the fiscal policy in the UK is not sustainable in the sense of satisfying of intertemporal budgetary constrain.

Keyword  Public debt – Budget deficit - Time varying parameter - Intertemporal constraint - Sustainability

JEL Classification: C22 - E62 - H63

1. Introduction
In recent years, the increased budget deficit and public debt experienced by many developed and developing countries have led to the increasing importance of government finance in the long run. Most countries face extensive pressure on their public debts and deficits in the last two decades. Due to the recent financial crisis and the outstanding fiscal stimulus to drive the world economy out of recession, it is important to satisfy the budget plan of government (Fincke and Greiner, 2011a).

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Especially European countries and US have suffered from very persistent problem by virtue of high public deficits and increase of the public debt. This situation describes vital problem from economic and political aspects, especially US and European countries both now and in future. Some fiscal rules are applied for European countries, limiting the public debt to GDP is 60% and public deficits to GDP is 3%. These measures have been established by Maastricht Treaty and where accepted for the European countries in 1992. In this context, the question is whether governments are able to respond in a sustainable way for persistent budget deficits and high public debt. Therefore, countries need to satisfy their deficits and debt in the short run, but they require that the present value of debt converges to zero asymptotically (Fincke and Greiner, 2011b). The government’s intertemporal budget constraint (IBC), within a dynamically efficient economy, requires fiscal policies to satisfy the present value borrowing constraint i.e., the present value of outlays equals the present value of revenues (Legrenzi and Milas, 2011).

This implies that the expected present value of expenditures (exclusive interest payments) must not exceed the expected present value of revenue, which is known as no-Ponzi game condition. The mean of this condition is that households cannot borrow and pay the interest payments with borrowing more (Greiner, Köller and Semmler, 2007). Sustainable way in the economy is related to the financial solvency of the government and is connected to the current and expected future fiscal and economic policies. If economic and fiscal agents expect the current and future fiscal and economic policies to cause intertemporal budget constraint (IBC), fiscal process is accepted to be sustainable and vice versa. The intertemporal budget constraint (IBC) in present value terms is also known as the present value budget constraint (PVBC) in US economics literature and the sustainability of public finance is vital for the long term (Bravo and Silvestre, 2002). If a given fiscal policy is estimated to be unsustainable, it has to transform a position in which the future primary balances are consistent with the budget constraints (Afonso, 2005). We follow the model developed by Bohn (1998) who has introduced an approach to calculate to the public debt for mean reverting process.

This approach is tested how the primary surplus relative to GDP responds to variations in public debt relative to GDP. Given the estimated positive response of primary surpluses to debt to GDP ratio, the government budget identity shows that the debt to GDP ratio must be mean reverting process. If the response of the primary surplus to the public debt is positive and statistically significant, the fiscal policy can be indicated to satisfy the intertemporal budget constraint of these governments.

The aim of our study is to test the validity of fiscal sustainability for US and UK over the period 1970-2012 through the approach by Bohn (1998) and the way it applied in Fincke and Greiner (2011b). The interest for US and UK is given by the fact that both countries experimented, with similar tendencies (except the period 1970-1982), especially in the last 10 years, an accentuate increase in the level of public debt, as Figure 1 illustrates. The main add of this paper is the estimation of time-varying parameter for US and UK, as very important signal for dynamic reaction of primary surplus to increases in public debt.

Fincke and Greiner (2011b) refer to three main reasons for the allowance of time-varying parameters in the regressions. The first reason shows that the true data generating process is unknown and most likely nonlinear. The nonlinear model is estimated based on time-varying coefficients (Granger, 2008), while the model with time-varying coefficients is more general than OLS estimation. The second reason highlights how react the government under public debt changes (see also Bajo-Rubio et al., 2004; Martin, 2000; Payne et al. 2008; or Westerlund and Prohl, 2010), while the last one reveals that the long-run coefficients are based on short-run coefficients through the level of random coefficients.
Figure 1 The public debt in US and UK, in the period 1970-2012

Source of data: IMF Historical Public Debt Database (September 2012)

We also consider that, according to Fincke and Greiner (2011b), the stock of real public debt is not adjusted through the seignorage or inflation. In this case, the central bank is independent and the government actions for reducing of public debt are not rely on central bank. Studies about these topics have devoted many efforts. In this area, the relationship between primary surplus and borrowing is especially examined in Table 1.

Table 1 Empirical results for public debt - primary surplus in the literature

<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Country - Period</th>
<th>Data Frequency</th>
<th>Methods</th>
<th>Sustainability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canzoneri; Cumby; Diba (2001)</td>
<td>US - 1951-1995</td>
<td>Annual</td>
<td>VAR</td>
<td>Yes</td>
</tr>
<tr>
<td>Greiner; Köller; Semmler (2007)</td>
<td>Italy, Germany, France, Portugal, US 1960-2003</td>
<td>Annual</td>
<td>OLS</td>
<td>Yes</td>
</tr>
</tbody>
</table>
The remainder of this paper is organized as follows. Section 2 lays out the theoretical framework and section 3 presents the empirical evidence and section 4 discusses the conclusion of this study.

2. Theoretical framework

The basic of this study focuses on the systematic relationship between the debt-to-GDP and the primary surplus, being largely based on the work of Fincke and Greiner (2011b). This relationship is examined for the first time by Bohn (1998), but it takes it basis from Barro’s (1979) tax-smoothing model that examines an effect of debt on primary surplus. Direct evidence from examining the positive response of the primary surplus to the debt-to-GDP ratio is enough a signal for sustainability. Because of some various shocks, such as fluctuations in income growth, interest rate and government spending, debt-to-GDP, ratio does not turn into the mean-reversion. In order to achieve the fiscal sustainability, it is noteworthy not only the positive response of primary surpluses to the debt-GDP ratio but also the satisfying the intertemporal budget constraint (Bohn, 1998). If government wants to reduce the stock of public debt, it can be appropriate to expect the government to reach the primary surpluses.

The main principle is to meet the debt stabilization and sustainable goals, so a positive response of primary balance to the stock of debt should be foreseen (Afonso, 2008; Bohn, 1998). The public debt has this form:

$$\frac{dB(t)}{dt} = r(t)B(t) + G(t) - T(t) = r(t)B(t) - S(t)$$

where $B(t)$ - the public debt, $r(t)$ - the interest rate, $G(t)$ - the public spending, $T(t)$ - the tax revenues, $S(t)$ - the primary surplus (including public revenues), and $t$ - the time.
The public debt is sustainable only if the intertemporal budget constraint is satisfied:

\[ \lim_{t \to \infty} B(t)e^{-\int_0^t r(t)dt} = 0 \]  
\[ \int_{t_0}^{\infty} e^{\int_{t_0}^t r(m)dm} S(t)d(t) \]  

(2)

Considering the primary surplus relative to GDP (S/Y), we can write:

\[ \frac{S(t)}{Y(t)} = a + b(t) \left( \frac{B(t)}{Y(t)} \right) \]

(3)

where \( \alpha \in \mathbb{R} \) - a constant, and \( \beta \in \mathbb{R} \) - a time varying reaction coefficient of primary surplus to public debt relative to GDP. On this framework, Fincke and Greiner (2011b) demonstrated that, for some period, the government reaction to the debt ratio may be zero or even negative, while in the average it must be positive. Otherwise, there is not any evidence for sustainable debt policy.

3. Empirical evidence

We apply a test that is based on the theoretical considerations of the last section to data for the US and UK during the period 1970-2012. Then, we analyse the correlation between the primary surplus and public debt all measured as ratios to GDP. Our empirical analysis employs the primary surplus and debt to GDP data provided by the US and UK and data is taken from the AMECO (Annual Macro-economic Database) and IMF Historical Public Debt Database (September 2012) over the period 1970 to 2012. These countries have been selected according to the data available. For the US and UK we estimate the reaction of the primary surplus to public debt, relative to GDP respectively, and present the results. The next step is to check the existence of mean reversion of public debt, testing the positive reaction coefficient.

In this case, we follow this equation (annual data):

\[ s(t) = b(t)b(t) + a^7 Z(t) + e(t) \]

(4)

where \( s(t) \) - the primary surplus as percentage of GDP, \( b(t) \) - the public debt as percentage of GDP, \( Z(t) \) - the vector of variables (which includes 1 in its first element, for the intercept, and additional variables in its other elements, having the influence on primary surplus ratio), \( e(t) \) - the error term (assumed to be i.i.d. N(0, \( \sigma^2 \))) and, finally, \( t \) - the time. \( Z(t) \) is entered according to Bohn (1998) and is relied on tax smoothing hypothesis. The hypothesis states that the public deficits should be used keeping the tax rates constant in order to minimize the excess tax burden. The regular inputs will finance the normal expenditures, while any deficits will cover the unexpected outputs. In this case, we will enter a business component - \( YVar \), which capture the fluctuation in revenues, by subtracting the long-term trend of GDP, with the Hodrick-Prescott-Filter (HP-Filter) to the real GDP series, from its actual values.

As the deviations of real public expenditures from its long-run trend affect the primary surplus, we inserted a second business cycle variable - \( GVar(t) \), by using the same HP-Filter. This component measures the fluctuations of public expenditures around its trend. Further, in order to deal with the endogeneity, we follow the contribution of Fincke and Greiner (2011b), by considering the lagged debt ratio \( b(t-1) \):

\[ s(t) = a_0 + b(t)b(t-1) + a_4 GVar(t) + e(t) \]

(5)
We estimate time-varying coefficients by using the penalized spline estimation (see also Hasti and Tibshirani, 1999, or Ruppert et al., 2003 among others), which is more robust than OLS estimation. Thus, we estimate the reaction coefficient $\beta(t)$ in equation (5) as a function of time. Firstly, we estimate equation (5) for USA and UK, for the data from 1970 until 2012. The result of the estimation is presented in Table 2.

Table 2 Coefficients for equation (5) for with data from 1970-2012

<table>
<thead>
<tr>
<th>Country</th>
<th>USA</th>
<th>UK</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coefficient</td>
<td>Stand. error (t-stat)</td>
</tr>
<tr>
<td>Constant</td>
<td>-1.367e-03</td>
<td>2.371e-02 (-0.058)</td>
</tr>
<tr>
<td>$b(t - 1)$</td>
<td>-1.743e-04</td>
<td>4.888e-04 (-0.357)</td>
</tr>
<tr>
<td>$GVar(t)$</td>
<td>5.024e-06</td>
<td>6.345e-06 (0.792)</td>
</tr>
<tr>
<td>$YVar(t)$</td>
<td>6.617e-05</td>
<td>7.347e-06 (9.007)</td>
</tr>
<tr>
<td>$sm(t)$</td>
<td>edf: 7.176</td>
<td>F: 35.71 p-value: 2e-16</td>
</tr>
</tbody>
</table>

$R^2$(adj): 0.918 DW: 1.282404

$R^2$(adj): 0.902 DW: 1.852696

Source: Authors’ calculation

The estimation outcome shown in Table 2 demonstrates that for the USA the average reaction coefficient of the primary surplus relative to GDP to variations in the debt to GDP ratio takes a negative value of 1.743e-04 percent and is statistically insignificant even at the 10 percent level. This implies that the USA government did not raise the primary surplus as government debt increased rather reduced it. However, this reduction has been insignificant. It can also be realized that the time-varying component of the reaction coefficient, denoted by $sm(t)$ in the 5th row, is statistically significant, too, and the estimated degrees of freedom (edf) of about 7.176 indicate that the reaction coefficient is not constant but a time-varying function.

However, for the UK the average reaction coefficient of the primary surplus relative to GDP to variations in the debt to GDP ratio takes a negative value of 0.0030855 percent and is statistically significant at the 5 percent level. This implies that the UK government did not raise the primary surplus as government debt increased rather reduced it and this reduction has been significant. It can also be realized that the time-varying component of the reaction coefficient, denoted by $sm(t)$ in the 5th row, is statistically significant, too, and the estimated degrees of freedom (edf) of about 9.275 indicate that the reaction coefficient is not constant but a time-varying function.
In Figure 2 are illustrated the graphics of the smooth term, for both countries. They are drawn such that a value of zero, for the smooth term, implies that the coefficient of reaction just equals its average value. At certain point in time, the actual value of the reaction coefficient equalises the average value plus the level of smooth term.

**Figure 2** The following figures present plots of the smooth term sm(t) that gives the deviation of the reaction coefficient from its average value.

The reaction coefficient has a constant tendency until 2000 in both countries, but after that, it dramatically declines. Starting to the year 2009, it registers a steeply increasing trend in UK relative to USA. In this case, the UK government does not put pressure on stabilization of public debt. This also shows there is an almost monotonously rising debt ratio to GDP over the considered period. As the coefficient of $GVar$ is positive, the primary surplus increases when public spending is above its trend. At the same time, primary surplus declines when GDP is below its trend because the coefficient of the variable $YVar$ is also positive. In both cases, the adjusted $R^2$ is around of 90 percent, indicating a good overall fit of the model. The Durbin-Watson test statistics (DW) does not suggest any issue in respect to the residuals autocorrelated, particularly for UK.

4. Conclusions

The US and UK are two countries, which have experimented in the last years significant budgetary deficits and also high level of public debts. In this context, we analyse whether US and UK have followed sustainable debt policies during the period 1970-2012. In order to test these aspects, we investigate the reaction of the primary surplus as percentage of GDP to variations in the debt to GDP ratio, as a powerful test. The main results reveal that the coefficient for UK is negative and significant. For the case of US, we are unable to find a clear-cut evidence of the sustainability of public debt (i.e. the coefficient is also negative, but insignificant).

In the case of the UK, the outputs reveal that government did not raise the primary surplus as the government debt increased rather reduced it and this reduction has been significant. Moreover, the significance of the reaction coefficient demonstrates that the reaction of the primary surplus to increases in public debt is not constant; instead, it varies over time. All these evidences allow us to appreciate that the fiscal policy in the UK is not sustainable in the sense of satisfying of intertemporal budgetary constrain.
The main policy implications suggest that it is required for the UK government to reduce the public expenditures, without any significant tax augmentation, in order to sustain the rise of public debt. More precisely, the authority should stimulate the primary surplus for covering the public debt in order to ensure the fiscal policy sustainability.

References

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