A Comparison of Ranking Criteria: an Application to Asset Class Indices of Europe, US, Russia and China

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Abstract The main purpose of this work is the ex post comparison of the performances of three macro asset class indices for Europe, United States, Russia and China during the period 2003-2015. The analysis is based on six different ranking criteria, starting with the well-known Sharpe index and its VaR and CVaR modifications, then considering the Omega and Sortino ratios which employ higher partial moments, and at last the Rachev ratio which changes the profitability index. All the previous performance indices give substantially the same ranking; they all show supremacy of Chinese fixed income and both European and American real estate in the years 2003-2008; they indicate US and European stock indices as the worst performers in 2008, and American, Russian and Chinese fixed income indices from 2012 to 2015 again as the worst performers. A visual display of the six different rankings is also provided, highlighting the adequacy of the Sharpe ratio against its more refined alternatives.

Keywords Performance measures - Sharpe ratio - Asset class ranking - Skewness - Kurtosis

JEL Classification G32 - F3

1. Introduction

Usually, portfolio or fund performance evaluation is based on reward to risk ratios. The classical measure is the Sharpe index (1966, 1994), that is the ratio between the excess return of the portfolio (with respect to the riskless asset return) and the return standard deviation. This means that the Sharpe index summarizes the portfolio characteristics in terms of mean and variance, which is only justifiable with normally (or, more in general, elliptically) distributed returns. This obviously represents the weakness of the Sharpe ratio and has led to the development of several new performance ratios, ultimately based on more accurate risk measurement and also on new profitability measures. In this paper we compare the ranking of 12 indices over the period 2003-2015 using 6 different reward to risk ratios.

The rest of the paper is divided into three sections. Section 2 is devoted to the description of the performance ratios used in our analysis. Section 3 describes the data set, the results
obtained in terms of ranking correlation and the comparison of the asset class performances over
the time. The last Section contains conclusions and comments.

2. Performance Measure

2.1 The Sharpe Ratio

The classical Sharpe ratio is given by:

\[
SR = \frac{E_P - r_f}{\sigma_p},
\]

where \( r_f \) is the observed return on a riskless asset, over a fixed period, while – from an \textit{ex post}
perspective – \( E_p \) and \( \sigma_p \) represent, respectively, the average of the \( N \) historically observed
portfolio (or index or asset) returns and their standard deviation:

\[
E_P = \frac{1}{N} \sum_{h=1}^{N} r_h,
\]

\[
\sigma_p = \left( \frac{1}{N-1} \sum_{h=1}^{N} (r_h - E_p)^2 \right)^{1/2}.
\]

2.2 The Reward to VaR Ratio

Dowd 2000 and Alexander and Baptista 2003 suggest the adoption of \textit{Value at Risk (VaR)} instead
of standard deviation as risk measure. We recall that, given a fixed low probability (1-c), the
\( \text{VaR}_{1-c} \) represents the opposite of the level of return such that there is a (1-c) probability the
random return realizations will fall under it. This formally gives the following formula for \( \text{VaR}_{1-c} \):

\[
\text{VaR}_{1-c} = -F_R^{c^-} (1-c),
\]

where \( F_R^{c^-} \) is the generalized (1-c)-quantile of the random return \( R \). Starting from \( N \) historical
data, \( \text{VaR}_{1-c} \) can be easily found considering the ordered set of realizations and choosing the value
corresponding to the \((1-c) \times N\) position.

With the previous notations, the reward to \( \text{VaR}_{1-c} \) ratio is:

\[
\text{VaRR} = \frac{E_P - r_f}{\text{VaR}_{P,1-c}}.
\]

2.3 The Reward to CVaR Ratio

Another improvement is due to Agarwal, Naik (2003) and Rachev \textit{et al} (2009): they introduce
the Conditional Sharpe ratio or \textit{STARR} ratio using \textit{Conditional Value at Risk (CVaR)}. \( \text{CVaR}_{1-c} \)
is
the opposite of the expected value of return \( R_p \), conditional to the values of return below \(-\text{VaR}_{1-c}\)
(mean restricted to return worst values), which translates to the following formula:

\[
\text{CVaR}_{1-c} = -E(R_P / R_P < -\text{VaR}_{1-c}).
\]
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Starting from historical data, \( CVaR \) can be evaluated as the opposite of the arithmetic mean of the observed values smaller than \( -VaR \), and the Conditional Sharpe ratio is:

\[
CVaRR = \frac{E_P - r_f}{CVaR_{P,1-c}}.
\]

### 2.4 Omega and Sortino Ratio

From the point of view of an investor or a fund manager, realizations above a target level of return \( \tau \) are good, while those below \( \tau \) are bad. This leads to considering higher partial moments (HPM) as profitability indicators and, on the contrary, lower partial moments (LPM) as risk indicators, where the higher and lower partial moments of order \( n \) are given by:

\[
\begin{align*}
HPM_n(\tau) &= \left[ \frac{1}{N} \sum_{h=1}^{N} \max(r_h - \tau, 0)^n \right]^{1/n}, \\
LPM_n(\tau) &= \left[ \frac{1}{N} \sum_{h=1}^{N} \max(\tau - r_h, 0)^n \right]^{1/n}.
\end{align*}
\]

This allows for a wide variety of performance ratios of the kind HPM/LPM, which differ for the degree of the partial moments, that shall be chosen accordingly to the level of investor risk aversion. Here we consider two famous ratios, the Omega (see Shadwick, Keating 2002) and the Sortino ratio 1991, which are as follows:

\[
\begin{align*}
Omega &= \frac{HPM_1(\tau)}{LPM_1(\tau)} = \frac{E_P - \tau}{LPM_1(\tau)} + 1, \\
Sortino &= \frac{E_P - \tau}{\sqrt{LPM_2(\tau)}}.
\end{align*}
\]

### 2.5 The Rachev Ratio

In order to refine the reward measure, new versions of performance ratios have been proposed (Rachev 2003, Biglova et al 2004). Rachev suggests to consider a “good” \( CVaR \), that is the expected value of return, conditional to the values of return greater than \( VaR_{1-c} \), as profitability index. The Rachev ratio can be expressed in the following way:

\[
RR = \frac{E(R_P / R_P > VaR_{P,1-c})}{CVaR_{P,1-c}}.
\]
3. Empirical Results

3.1 Data and Main Statistics

Monthly data from Bloomberg\(^1\) database have been used. We have considered three main asset classes, namely equities, fixed income and real estate. The historical quotes are referred to the period 2003 - 2015 and pertain to 12 indices, relative to Europe, US, Russia and China. More in detail, in the numerical application we used:

- STOXX Europe 600 Price Index EUR (Bloomberg ticker: SXXP Index): a derivation of the STOXX Europe Total Market Index that has 600 components and represents large-, mid- and small-capitalization companies across 18 European countries;
- S&P 500 Index (SPX): a capitalization-weighted index of 500 US stocks;
- MSCI Russia Index (MXRU): a float-weighted equity index that captures the performance of the large- and mid-capitalization segments of the Russian market, covering approximately 85% of the float-adjusted market capitalization in Russia;
- Hong Kong Hang Seng Index (HSI): a float-weighted equity index of a selection of companies from the Stock Exchange of Hong Kong;
- EUG5TR Index: a Bloomberg/EFFAS (European Federation of Financial Analysts’ Societies) long term European government bond index;
- USG5TR Index: the analogous of the EUG5TR index for US;
- Russian Government Bond Index (RGBI): a weighted index of Russian government bonds;
- FGGYCN1 Index: a FTSE index of medium- and long-term Chinese government bonds;
- Bloomberg Europe 500 Real Estate Index (BERALE): a capitalization-weighted index of all companies that are in the real estate sector of the Bloomberg Europe 500 Index;
- Bloomberg NA REITs (BBREIT): a weighted index of US Real Investment Trusts with capitalization not less than $15 millions;
- Russia Housing Prices New Apartments (RUPHNRF): developed by the Russian Federal Service of Statistics, this index summarizes changes in residential property prices;

For each of these indices, we evaluated the logarithmic return and computed the estimation of its moments, obtaining for skewness a minimum value equal to -2.59 and a maximum equal to 2.39, while for the excess kurtosis a minimum of 1.13 and a maximum of 10.75 (excluding Russian real estate, since it is quoted quarterly and presents excess kurtosis anomalies). High values in both skewness and kurtosis give evidence of a distribution far from normality for the observed return. In evaluating indices we have used a null risk-free rate \(r_f\) and a null target level of return \(\tau\). For \(\text{VaR}\) and \(\text{CVaR}\) in \(\text{VaRR}\), \(\text{CVaRR}\) and Rachev ratio, a value of \(c\) equal to 99% has been used, while for the good \(\text{CVaR}\) in the Rachev ratio \(c_1\) is taken equal to 50%.

3.2 Ranking Correlation

In order to quantify the relation between the performance measures, we have evaluated the Spearman rank correlation coefficients year by year. All performance measures, except for Rachev ratio, display usually good rank correlation with respect to the Sharpe ratio, between 0.028 (\(\text{CVaR ratio, 2015}\)) and 0.979 (Omega ratio, 2013). Table 1 and 2 report the Spearman rank correlation coefficients.

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\(^1\) Source: Bloomberg Finance L.P. Closing adjusted prices have been considered. The quotes of the Russian real estate index are given only quarterly, all the others are quoted daily, weekly, monthly and so on.
correlation coefficients related to the year with the minimum (2015) and the maximum (2013). It should be noted that the CVaR ratio in 2015 shows a low level of correlation with any of the other performance ratios, which is probably due to the scarce amount of historical data considered in the CVaR evaluation. On average, the rank correlation of the Sharpe ratio in relation to the other examined performance measures over the years amounts to 0.642.

Table 1

<table>
<thead>
<tr>
<th>2015</th>
<th>SR</th>
<th>VaRR</th>
<th>CVaRR</th>
<th>Omega</th>
<th>Sortino</th>
<th>Rachev</th>
</tr>
</thead>
<tbody>
<tr>
<td>SR</td>
<td>1</td>
<td>0.811</td>
<td>0.028</td>
<td>0.804</td>
<td>0.741</td>
<td>0.783</td>
</tr>
<tr>
<td>VaRR</td>
<td>1</td>
<td>0.056</td>
<td>0.993</td>
<td>0.923</td>
<td>0.958</td>
<td></td>
</tr>
<tr>
<td>CVaRR</td>
<td>1</td>
<td>0.077</td>
<td>0.014</td>
<td>0.014</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Omega</td>
<td></td>
<td>1</td>
<td>0.951</td>
<td>0.972</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sortino</td>
<td></td>
<td></td>
<td>1</td>
<td>0.951</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rachev</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2

<table>
<thead>
<tr>
<th>2013</th>
<th>SR</th>
<th>VaRR</th>
<th>CVaRR</th>
<th>Omega</th>
<th>Sortino</th>
<th>Rachev</th>
</tr>
</thead>
<tbody>
<tr>
<td>SR</td>
<td>1</td>
<td>0.951</td>
<td>0.951</td>
<td>0.972</td>
<td>0.979</td>
<td>0.951</td>
</tr>
<tr>
<td>VaRR</td>
<td>1</td>
<td>0.888</td>
<td>0.979</td>
<td>0.951</td>
<td>0.986</td>
<td></td>
</tr>
<tr>
<td>CVaRR</td>
<td>1</td>
<td>0.937</td>
<td>0.909</td>
<td>0.895</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Omega</td>
<td></td>
<td>1</td>
<td>0.951</td>
<td>0.979</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sortino</td>
<td></td>
<td></td>
<td>1</td>
<td>0.930</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rachev</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In general, there is also a good correlation between the other pairs of performance ratios, except sometimes for the Rachev ratio and the VaR ratio. Our analysis partially confirm the results obtained by Eling, Schuhmacher 2007, who find a very high rank correlation of the examined performance measures with respect to the Sharpe ratio and also in relation with each other.

3.3 The Best Asset Classes

In the previous paragraph we have found similarities in all the rankings. A refined analysis, using weekly or daily data will probably give more precise answers and results strictly close to those shown by Eling and Schuhmacher. This means that different performance indices can be considered as thermometers calibrated with different degrees, but they give the same ranking. In order to give a visual representation of this result and a more immediate analysis, we have used Excel contour charts. Year by year, the white areas cover the two best performing asset classes (the top), the grey areas represent the five medium performing asset classes, while the five worst performing asset classes are painted in black. Hereafter we report the graphs for every performance indicator.
The graphs may seem very different, but there are interesting similarities that are worthy of note. For example, the “black snake” on the right (in Picture 1(a) the detail from the Sortino graph is everywhere evident, even though less so in the CVaR ratio). This shows that from 2012 to 2015 American, Russian and Chinese fixed income indices are the worst performers, whatever performance measure we choose.

Another shared feature, except for CVaR ratio, is the “giraffe” on the bottom left, shown in Picture 1(b). This can be interpreted as the supremacy of Chinese fixed income and European and American real estate in the years 2003-2008.

Lastly, the black trapezoid in the upper part of almost all graphs corresponding to the year 2008 suggest US and European stock indices as the worst performers in that period, which we can easily relate to the financial crisis.

**Picture 1** Details from the Sortino graph: (a) the black snake; (b) the white giraffe.

![Picture 1](image)

**4. Conclusions**

Performance measures are fundamental in the analysis of the behaviour of a portfolio, an asset or an index. On one hand, they are key to evaluate *ex post* whether or not a fund manager has met their purposes or, better, those of the client. On the other hand, performance ratios can find application in optimal portfolio selection: starting from the assumptions on the portfolio return distribution and maximizing *ex ante* the selected performance ratio.

In our paper we analyzed and compared different *ex post* performance ratios of three major asset classes for four areas (Europe, US, Russia and China) over the period 2003-2015. Our analysis gives evidence of a relationship between the rankings obtained using different performance indices.

This suggest the opportunity to rely on the simplest and widely used Sharpe ratio, despite the criticism vastly evidenced in literature.

We have used a visual representation of the results that allows a more immediate comparison of the adequacy of the performance ratios in evaluating the financial landscape during the period 2003-2015: all the performance indices seem to be able to capture the past decade evolution. This confirms that, given an appropriate stochastic modelling of the return dynamics, they can be usefully applied *ex ante* in portfolio choice.
References


