

Variable Annuities: New Solution to Long-Term Investment Problem

Ludovic Goudenège • Andrea Molent • Antonino Zanette

Abstract In this article we provide an introduction to a class of insurance product called Variable Annuities and to pricing methods that can be used to evaluate them. These products have been present for a few years on the American, European and Asian markets, and continue to capture the attention of new investors. They could become the solution to retirement problems even in countries like China and India that, after the booming economy, will now face the social security problem of the new middle class.

Keywords Variable Annuities - Insurance - Risk Management - Pricing - Hedging

JEL Classification G2 - G22 - G12 - G13 - C6

Introduction

In 2008 following the subprime crisis, financial markets have suffered the upheavals that have affected the entire world economy. Other events have recently rocked the financial markets: in 2015 the slowdown of Chinese growth and in 2016 the Brexit. Since the subprime crisis, these markets were extremely volatile: this situation persists and perhaps became the new standard. After many failures, the gap between the different interest rates applied to different transmitters has become larger and larger and a discussion on the identification of the risk-free rate is open. The ECB and the Fed's rates gradually declined, while the rate on sovereign debt increased gradually, and then dropped. For customers, it is difficult to balance risk and return. In this context, clients seek protection for their savings, and the ability to take advantage of the positive changes in the market. With regard to the social problematic, following the increase in life expectancy, standard annuities for retirement dropped. The mission of insurance companies is to answer the request for protection and compensation of their customers. The solution is to provide the customer an investment account and cover its value with guarantees.

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These products are called Variable Annuities. In the words of François Robinet, CEO of AXA Life Invest, “These products, unit of account guaranteed, will become a solution to solve the long-term investment problems with security, and prepare for retirement”. Variable Annuities are insurance life contracts in account units with guaranteed revenues or capital. They were launched by AXA in the United States in 1995, and appeared in Italy in 2008. Variable Annuities are mainly diffused in USA, Japan, and North Europe; they are attractive products especially with the retirement reforms and the new sales may reach \$22 billion by 2018, with a 57% increase from 2012 (Think Advisor, 2014). Variable Annuities are nevertheless exotic products with hidden options (the main one is the lapse option) and two kinds of risk: the market risk and the actuarial risk. Because of these characteristics, it is difficult to price these new kinds of optional products, and manage the ingrained risks.

This paper is organized as follows. In Section 1 we present an overview of Variable Annuities (definition, properties, guarantees and fees). In Section 2 we present some example of Variable Annuities sold in different countries (Europe, US, and Asia). In Section 3 we focus on Risk Management, and Pricing (risks associated with these products and pricing methods).

1. Variable Annuities: an overview

The term Variable Annuity is used to refer to a wide range of life insurance products, whose benefits can be protected against investment and mortality risks by selecting one or more guarantees out of a broad set of possible arrangements. Despite the unique characteristics can change, there are some common to all of them: a Variable Annuity is a long-term, tax-deferred investment, designed for obtaining a post-retirement income. Formally, a Variable Annuity is a contract between a policyholder and an insurance company, under which the insurer agrees to make periodic payments to the policyholder beginning either immediately or at some future date. The holder buys a Variable Annuity contract by making either a single purchase payment (lump sum) or a series of purchase payments (accumulation phase).

Variable Annuities were introduced in the 1970s in the United States and in the first years of 2000, they became popular in Europe (especially Germany, UK, and France) and known in other European countries such as Italy: see Bonafede (2008). In the late 1990's Variable Annuities were introduced by insurance companies in Asia and have been popular ever since: see Summit (2012). The cause of the success of these policies is that they offer a range of investment options. The value of the investment will change depending on the performance of the investment options chosen. The investment options for a Variable Annuity are typically mutual funds that invest in stocks, bonds, money market instruments, or some combination of the three.

Variable Annuities are designed to be long-term investments, to meet retirement and other long-range goals. Despite the ingrained guarantees, these products involve investment risks, just as mutual funds do. Although Variable Annuities are typically invested in mutual funds, they differ from mutual funds in several important ways. First, Variable Annuities may let the policyholder receive periodic payments for the rest of his life (or the life of any other person designated). This feature offers protection against the possibility that, after retirement, the policyholder will outlive his assets.

Second, Variable Annuities may have a death benefit. If the policyholder dies before the insurer has started making payments, the beneficiary is guaranteed to receive a specified amount - typically at least the amount of the purchase payments. The policy beneficiary will also get a benefit from this feature if, at the time of the death of the policyholder, his account value is less than the guaranteed amount.

Third, Variable Annuities are tax-deferred. That means the policyholder pays no taxes on the income and investment gains from his annuity until he withdraws his money. He may also transfer his money from one investment option to another within a Variable Annuity without paying tax at the time of the transfer. When the policyholder takes his money out of the Variable Annuity, however, he will be taxed on the earnings at ordinary income tax rates (for example, according to Italian Law, the financial return, equal to the difference between the amount paid and the premiums paid, shall be subject to the application of a substitute tax on income, at the time of payment of the benefit, according to what provided by the D.L. August 13, 2011 n. 138, converted into Law 148 of 14 September 2011). For more information, see the webpage (Formula Accumulation).

A Variable Annuity has usually two phases: an accumulation phase and a payout phase. During the accumulation phase, the policyholder makes purchase payments, which he can allocate to a number of investment options. For example, he could designate 40% of his purchase payments to a bond fund, 40% to a U.S. stock fund, and 20% to an international stock fund. The money he has allocated to each mutual fund investment option will increase or decrease over time, depending on the fund's performance. In addition, Variable Annuities often allow the policyholder to allocate part of his purchase payments to a fixed account. A fixed account, unlike a mutual fund, pays a fixed rate of interest. The insurance company may reset this interest rate periodically, but it will usually provide a guaranteed minimum (e.g., 1% per year). During the accumulation phase, the policyholder can typically transfer his money from one investment option to another without paying tax on his investment income and his accumulation phase, but in case of lapse, he may have to pay "surrender charges" to the insurance company, which are discussed below. At the beginning of the payout phase, the policyholder may receive his purchase payments plus investment income and gains (if any) as a lump-sum payment, or he may choose to receive them as a stream of payments at regular intervals (generally monthly). If he chooses to receive a stream of payments, he may have a number of choices of how long the payments will last. Under most annuity contracts, he can choose to have his annuity payments last for a period that he sets (such as 20 years) or for an indefinite period (such as his lifetime or the lifetime of him and his spouse or other beneficiary). During the payout phase, policyholder's annuity contract may permit him to choose between receiving payments that are fixed in amount or payments that vary based on the performance of the mutual fund investment options. The amount of each periodic payment will depend, in part, on the time period that the policyholder selects for receiving payments. Some annuities do not allow withdrawing money from the account once the policyholder has started receiving regular annuity payments. In addition, some annuity contracts are structured as immediate annuities, which means that there is no accumulation phase and the policyholder will start receiving annuity payments right after he purchases the annuity. Unlike the with-profit or participating business, reference funds backing Variable Annuities are not required to replicate the guarantees selected by the policyholder, as specific assets hedge these ones. Therefore, reference fund managers have more flexibility in catching investment opportunities. One of the peculiarities of Variable Annuity policies is not making use of traditional financial management, but using of hedging techniques - called the "dynamic hedging" - thanks to which the insurance company has no direct impact on the asset allocation funds. Through these programs, it is therefore always possible to rebalance its market position following the changes in benchmark indices which allow to counteract their liabilities, but without directly engaging the fund of the policy. Therefore, the guarantee is external to the fund and does not affect the asset allocation. For this reason, Variable Annuity products may be defined multidimensional guaranteed products as opposed to traditional unit-linked products that can be considered one-dimensional structures.

Guarantees of performance associated with unit-linked products of the first generation are normally made through the annexation of a guaranteed fund to the family of funds available for the product concerned. The financial guarantee (of performance, capital etc.) is then “included” in specific funds with assets specifically chosen to cope with the guarantee offered by the funds themselves.

Instead, the return guarantees associated with Variable Annuities are options associated to the contract: they are additional features that policies designer add to the contracts. At each fund (not guaranteed) and at each benefit is associated a cost: the total cost is theoretically equal, for each fund selected by each insurance position, to the sum of the cost of the fund and of the guarantees associated with it. Variable Annuities typically offer a greater growth potential and greater risk than a fixed annuity. That is because, with a Variable Annuity, holders are able to select from a variety of professionally managed investment options, which may contain stock and/or bond portfolios. The account value is directly exposed to market variations, but the policy value is protected by guarantees.

The Death Benefit and other features

A common feature of Variable Annuities is the death benefit. If the policyholder dies, a person he selects as a beneficiary (such as his spouse or child) will receive the greater of: all the money in the policyholder account, or some guaranteed minimum (such as all purchase payments minus prior withdrawals). This second case is known as GMDB (Guaranteed Minimum Death Benefit). Some Variable Annuities allow the policyholder to choose a “stepped-up” death benefit. Under this feature, the guaranteed minimum death benefit may be based on a greater amount than purchase payments minus withdrawals. For example, the guaranteed minimum might be the account value at a specified date, which may be greater than purchase payments minus withdrawals if the underlying investment options have performed well. The purpose of a stepped-up death benefit is to “lock in” the investment performance and to prevent a later decline in the value of the account from eroding the amount that the holder expects to leave to his heirs. This feature carries a charge, however, which will reduce the account value.

Variable Annuities sometimes offer other optional features, which also have extra charges. The policyholder pays for each benefit provided by his Variable Annuity. These charges are usually independent and tied to the relative benefit. The following are the most common optional features. For more details, see Bacinello, Millosovich, Olivieri, and Pitacco (2011).

- GMAB (Guaranteed Minimum Accumulation Benefit): this feature provides to guarantee the minimum amount received by the annuitant after the accumulation period, protecting the value of the annuity and the annuitant from market fluctuations. The GMAB will be used only if the market value of the annuity is below the minimum guaranteed value.
- GMIB (Guaranteed Minimum Income Benefit): a common feature that guarantees a particular minimum level of annuity payments, even if the policyholder does not have enough money in his account (perhaps because of investment losses) to support that level of payments. When the annuity has been annuitized (start the paying of the annuities), this specific option guarantees that the annuitant will receive a minimum value’s worth of payments.
- GMWB (Guaranteed Minimum Withdrawal Benefit): this specific option gives annuitants the ability to protect their retirement investments against downside market risk by allowing the annuitant the right to withdraw a maximum percentage of their entire investment each year until the initial investment amount has been recouped. The GMWB is the real novelty of Variable Annuities in respect of traditional life insurance contracts.

These three options are also called GMxBs guarantees (namely, Guaranteed Minimum Benefits of type 'x'). Another Variable Annuity adds to the previous three:

- GLWB (Guaranteed Lifelong Withdrawal Benefit): this option is similar to GMWB, but this policy has no fixed maturity. The annuitant has the right to perform periodic withdrawals, with a minimal guaranteed withdrawal, for all his life. A priori, there are no limits on guaranteed withdrawals, and on the total guaranteed withdrawal. Usually, a death benefit is always included.

These contracts may include other features such as long-term care insurance (LTC), which pays for home health care or nursing home care if the policyholder becomes seriously ill. The most common forms of guarantees associated with the growth of the benefit base are roll-ups: this is the simplest form of return guarantee. A roll-up provides guaranteed appreciation of the benefits base at a specific interest rate. The guarantee may accrue on a simple or compound interest basis. A 0% roll-up is the same as a return-of-principal guarantee.

Some products include ratchets, also called a "high watermark." With a ratchet, the benefits base is set equal to the highest of all values of the underlying funds throughout the accumulation phase, evaluated at a pre-defined time interval (e.g. annually). At various frequencies the existing benefits base is compared to the account value, and if the account value is higher, the benefits base is "ratcheted" up to the new level.

Another feature that may be included in some products are resets: resets are triggered at the discretion of the policyholder. They involve a comparison of the current account value to the original account value, and the benefits base is reset to the higher level. Other policy provisions such as a waiting period may be reset as well. Finally, some Variable Annuities offer guaranteed appreciation of the benefits base that combines one or more of the above forms of guarantees. For example, a common combination guarantee is the maximum of a roll-up and a ratchet. For example: a man owns a Variable Annuity that offers a death benefit equal to the greater of account value or total purchase payments minus withdrawals. He has made purchase payments totaling 100000 €. In addition, he has withdrawn 10000 € from his account. Because of these withdrawals and investment losses, his account value is currently 80000 €. If he dies, his designated beneficiary will receive 90000 € (the 100000 € in purchase payments he put in, minus 10000 € in withdrawals).

Charges and other costs

Variable Annuities include many charges that the policyholder has to pay. The insurance company withdraws most of them from the holder account. Some of these charges are used to hedge the policy guarantees, while other are used to cover administrative expenses. These charges will reduce the value of the account and the return on policyholder investment. The insurance company uses the guarantee charges to cover the guarantees of the policy. They are a fixed percentage of the account value, and are usually withdrawn continuously. These charges are active for the whole product life and are fixed at the beginning in the contract; finding the fair value of these fees, consists in pricing the product.

For example, the guarantee charges of a Variable Annuity are 2%. If the initial account value is 10000 €, and the linked fund increases of 5%, then the final value of the account value is . Usually, the policyholder can withdraw money from his policy within certain limits; usually this upper bound is a maximum sum of money per period. The contract may provide this upper limit to be exceeded by the holder. In this case, he will have to pay a penalty for the overdraft. The policyholder may also decide to withdraw the maximum possible and terminate the contract. In this case, he takes the value of the account reduced by a given percentage in addition to fixed

administrative costs for an anticipated recession. These expenses for contract termination are called surrender charges. These charges are partially used to pay a commission to his financial professional for selling the Variable Annuity. Generally, the surrender charges consist for the most in a percentage of the amount withdrawn, and declines gradually over a period of several years, known as the surrender period. For example, a 7% charge might apply in the first year after a purchase payment, 6% in the second year, 5% in the third year, and so on until the eighth year, when the surrender charge no longer applies. For example, let us suppose that a man purchases a Variable Annuity contract with a 10000 € purchase payment. The contract has a schedule of surrender charges, beginning with a 7% charge in the first year, and declining by 1% each year. In addition, the contract allows the policyholder to withdraw 10% of his contract value each year free of surrender charges. In the first year, he decides to withdraw 5000 €, or one-half of his contract value of 10000 € (assuming that his contract value has not increased or decreased because of investment performance). In this case, he could withdraw 1000 € (10% of contract value) free of surrender charges, but he would pay surrender charge of 7%, or 280 €, on the other 4000 € withdrawn.

Mortality and expense risk charge are equal to a certain percentage of the account value, typically in the range of 1.25% per year. This charge compensates the insurance company for insurance risks it assumes under the annuity contract. The profit from the mortality and expense risk charge is sometimes used to pay the insurer's costs of selling the Variable Annuity, such as a commission paid to the financial professional for selling the Variable Annuity. For example, a Variable Annuity has a mortality and expense risk charge at an annual rate of 1.25% of account value. The average account value during the year is 20000 € so the policyholder will pay 250 € in mortality and expense risk charges that year. The insurer may deduct charges to cover record-keeping and other administrative expenses. This may be charged as a flat account maintenance fee (perhaps 25 € or 30 € per year) or as a percentage of his account value (typically in the range of 0.15% per year).

Finally, special features offered by some Variable Annuities, such as a stepped-up death benefit, a guaranteed minimum income benefit, or long-term care insurance, often carry additional fees and charges. Other charges, such as initial sales loads, or fees for transferring part of the account from one investment option to another, may also apply.

Bonus credits

Some insurance companies offer Variable Annuity contracts with “bonus credit” features. These contracts promise to add a bonus to the contract value based on a specified percentage (typically ranging from 1% to 5%) of purchase payments. Variable Annuities with bonus credits may carry a downside; higher expenses can outweigh the benefit of the bonus credit offered. Frequently, insurers will charge the policyholder for bonus credits in one or more ways. Surrender charges may be higher for a Variable Annuity that pays a bonus credit than for a similar contract with no bonus credit; the policyholder purchases payments may be subject to surrender charges for a longer period than they would be under a similar contract with no bonus credit; higher annual mortality and expense risk charges may be deducted for a Variable Annuity that pays the holder a bonus credit. Although the difference may seem small, over time it can add up. In addition, some contracts may impose a separate fee specifically to pay for the bonus credit.

2. Examples from the international market

In this section, we present some real examples of Variable Annuities sold in different countries. The sophistication degree of these products is directly dependent from risk appetite and knowledge of financial markets of the country inhabitants.

European market

Variable Annuities are available in the many European countries and here we present some examples. For more information about Variable Annuities in Europe, see the webpage (Borsa Italiana). The percentage data refers to insurance products sold in Italy, and they may be different for other European countries. Many insurance companies compete on the Variable Annuity market. For example, we talk about AXA, with the product of the Accumulator line, Assicurazioni Generali, with Generali Premium and Allianz with “Allianz Invest4Life”.

The selection Accumulator by AXA offers GMAB products; for more details, see the webpage (La nostra offerta). For example, the Variable Annuity Accumulator Investimento offers a minimum return of 15.00% in 10 years in return for payment of an initial prize fund of at least 2500 €. If the policyholder keeps the investment after the expiry of the contract, it turns into a traditional unit-linked (so, by that date, there is no guarantee of capital) in the event of early redemption in the first four years instead the policyholder pays a penalty of 1.00%. The Variable Annuity “Accumulator Reddito” is instead a long-life product with accumulation guarantees. After at least five years from the effective date, the insurance policy provides, at the request of the policyholder and as long as he is alive, the chance to get a Performance Scheduled annual Guaranteed (PSG), which occurs through partial redemptions for a period of 20 years, equal to 5.00% of the greater of the premium paid capitalized at 2.50% compound annual rate of return for the first 5 years of the contract, and the value of the holder account upon receipt of the delivery request. A death benefit is included. The total fees paid by the policyholder is equal to 1.00%.

Generali Premium consists of a scheduled program of recurring single premiums, by installments (minimum 150 €), of the annual minimum of 1200 €; see the webpage (Generali Premium). It is possible to exercise the right of redemption if at least the top three premium annuities have been paid and the insured is alive. This product provides at the end of the accumulation phase (minimum 10, maximum 20 years) a 0.00% minimum guaranteed return. Generali Premium is characterized by the possibility of diversify the investment in more segregated funds and more internal funds; alternatively, the possibility to sign a savings plan linked to a segregated fund expressed in US dollars. The contract also includes a bonus at maturity. There is also coverage in the event of accidental death, doubling in case of road accident death (A.C.M.A -I.S.); the level of risk varies according to the investment allocation between the various segregated funds and any internal funds. A death benefit is included. This Variable Annuity offers a GMAB optional feature as AXA one.

Another product is Allianz Invest4Life by Allianz. It is a GLWB Variable Annuity type single-premium product, reserved for those who have already turned 45. The advantages of this product combine the opportunity to invest in the financial markets and the guaranteed minimum annuity. The contract provides for the payment of a minimum lump sum amount of 25000 €. Additional payments are not permitted. The disbursement of the annuity can be immediate or deferred, up to a maximum of 10 years. At the end of the accumulation period, the policyholder can choose the withdrawal frequency: monthly or yearly. This amount is determined by multiplying the premium paid by policyholder for conversion factor reported in the proposal form, which depends on the age of the insured at the time of subscription, and the period of deferment selected from policyholder: see Table 1 for an example of these coefficients. In addition to the annuity, the product offers insurance coverage in case of death insured. The contract provides for the coverage of commercial costs, a loading cost equal to 3.00% of the initial premium, a cost of hedging the guarantees amounting to 1.20% pa and an administration fee of 0.45% per annum. The contract does not include any cost of insurance coverage in case of death.

Table 1 The conversion factors for annual annuities of “Allianz Invest4Life” Variable Annuity

Age of the Policyholder	Duration of the deferral period		
	0 years	5 years	10 years
45	n.a.	2.41%	2.77%
50	2.31%	2.66%	3.11%
55	2.54%	2.97%	3.54%
60	2.83%	3.37%	4.13%
65	3.19%	3.91%	4.96%
70	3.65%	4.63%	n.a.
75	4.20%	n.a.	n.a.

The acronym “n.a.” means “not available”.

American Market

The United States is the country where the first Variable Annuities were designed and sold. The insurance companies that sell these products are many, and the products that are sold have features among the most varied.

As an example, we present FlexChoice Level by Metlife: see (Metlife FlexChoice). It is a GLWB contract where payments are different if the account value is positive or null. The contract includes a 5% roll up feature for the first 10 years, and an annual ratchet (Automatic Step-Up) that may increase the base benefit, as well as additional purchase payments. The annual charge is 1.20% per year, but may be increased up to 2.00% in case of an Automatic Step-Up. The product includes a death benefit guarantee and this guarantee costs each year 0.65% of the death benefit base. The policyholder must allocate the 80% of the account value in some funds stated by the contract (Protected Growth Strategy Portfolios) and the remaining 20% to one or more of the available more aggressive funds (Asset Allocation Portfolios). FlexChoice Level provides the policyholder with a level amount of guaranteed payments for his lifetime (Single Lifetime option) or for the life of the policyholder and his (her) consort (Joint Lifetime option). The guaranteed withdrawals are calculated as the product of the Base Benefit and of the Withdrawal Rate. Withdrawal Rate are shown in Table 2.

Table 2: The withdrawal rates for FlexChoice Level by Metlife

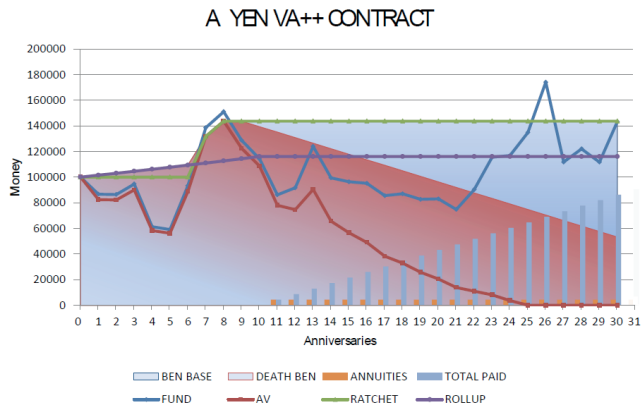
Age at 1st withdrawal	Before AV reduces to zero		After AV reduces to zero	
	Withdrawal Rate (% of Benefit Base)	Single Lifetime Guarantee Rate	Single Lifetime Guarantee Rate	Joint Lifetime Guarantee Rate
59 ½ to less than 65	4.00%	4.00%	4.00%	3.00%
65 to less than 70	5.00%	5.00%	5.00%	4.00%
70 to less than 80	5.25%	5.25%	5.25%	4.25%
80 +	5.75%	5.75%	5.75%	4.75%

Asian market

According to Insurance News Net’s “After financial crisis, Asia emerges as Variable Annuity growth area”; see Summit (2012). There is still a lot of potential for Variable Annuities growth in Asia even as the American Markets are increasing again after subprime crisis. In 2012, Rebecca

Ng of A.M. Best Company wrote an article about the potential for Variable Annuities growth in Asia and said that China looks to be the top spot for this growth; see Ng (2012). The rapid growth of the Japanese Variable Annuity market has begun after the Japanese financial deregulation from 1998 to 2001, which consisted of a series of measures to make it theoretically possible for Japanese, foreign institutions alike to perform banking, insurance, stock brokering, and investment services in yen or any other currency. The Variable Annuity product concept was unique, compared to the dominant traditional life products in the Japanese insurance market at that time, and has gained the interest of many investors. An emblematic example of Variable Annuity is the Yen Variable Annuity++ sold by AXA in the Japanese market.

Figure 1 An example of the development of a Variable Annuity contract (YENVA++)



It is a Variable Annuity with GMIB and GMDB, with both roll-up (up to 10 times) and yearly ratchet. There exist two versions of this product: a 25 years TC (time certain) and a whole life (WL). The premium is paid as a unique solution (single premium), and invested in an investment fund. The policyholder can lapse anytime with no surrender charges. The ratchets take place yearly, every anniversary date throughout the contract. Let AV_t be the value of the account linked to the fund. The initial income benefit of the ratchet is equal to the initial gross premium, and each year during the deferral period, it is updated as the max of the previous ratchet benefit and the actual account value. For the annuity payment period the ratchet benefit is updated in two different ways for the two product types. For the TC product we continue to use define it as in the deferral period, while for the WL product we have

$$IB_ratchet_t = \begin{cases} \max(IB_ratchet_{t-1}, AV_t + \sum_{s<t} Payout_s), & \text{if } AV_t > 0 \\ IB_ratchet_{t-1}, & \text{if } AV_t = 0 \end{cases}$$

The roll up benefit is calculated each year at the contract anniversaries, up to 10 years and only during the deferral period. The roll up rate is equal to 1.5% for the TC, and 2.5% for the WL. The initial roll up benefit is equal to the initial gross premium, and at each anniversary, the roll-up benefit is increased by a fixed roll-up rate. The GMIB is calculated as the max between the ratchet benefit and the roll-up benefit. At each anniversary, if the policyholder is still alive, he receives a sum equal to the product of the income benefit and a contract fixed ratio (for products sold in 2015, the factor is worth 3% for the WL, and the inverse of the number of payout phase years for the TC). The policyholder can chose the last of the deferral period at the beginning of the contract with some duration limits.

The death benefit is calculated as

$$DB_t = \max(AV_t, IB_t - \sum_{s < t} \text{Payout}_s)$$

There are several charges: at the entry, a 5% is calculated on the single premium paid. Everyday throughout the contract life, mortality, expense, and fund management charges are calculated as a percentage of the AV, and withdrawn. Figure 1 shows an example of the development of a YEN Variable Annuity++ WL contract.

Manulife (Singapore) introduced in 2007 *Secure Retirement Plus*, the first variable annuity product in Singapore; see the webpage (Retirement-insurance). Addressed specially to the territory's post-war "baby boomer" generation, Manulife Secure Retirement Plus offers a guaranteed withdrawal benefit that provides a regular stream of retirement income for at least 20 years (GMWB) or income for life (GLWB) from age 65, regardless of market performance. Unlike other annuity products, the GWB has growth potential through the Company's Loyalty Bonus and Step-Up mechanism. At a glance, Secure Retirement Plus provides an annual guaranteed withdrawal amount equal to 5% of the subscription for minimum of 20 years. The guaranteed retirement income is secure and will never go down, even in a declining market. A GMAB option may also be included: a 50% guaranteed Loyalty Bonus that upsizes the Guaranteed Withdrawal Benefit to 150 per cent of the initial investment if withdrawals are deferred in the first 10 years. An automatic step-up opportunity to lock in investment gains to the Guaranteed Withdrawal Benefit on every 5th policy anniversary up to age 80 is included. This could further enhance the stream of retirement income. Beginning at age 65, the policyholder can elect to make withdrawals that are guaranteed for the rest of life (GLWB). Investors have the flexibility to access their account value at any time or make switches from one portfolio fund to another. Finally, this product has a guaranteed death benefit: the heirs can chose between receiving the account value or the remaining regular withdrawals.

The Chinese insurance market is rapidly developing, and the various insurance companies struggle to get more land. For example, according to Thevenin (2015), AXA is accelerating in China to reach 100 million customers in Asia by 2030, and also the rival companies such as ICBC and Tian Ping show strong growth.

In June 2011, AXA-Minmetals and Sino-US United Metlife launched their first Variable Annuities on Chinese market; see Huang (2012). Both the two contracts provide for a single initial payment and include a GMAB. The maturity is respectively 7 and 10 years. The Guarantee Charges are around 1.5% of the Account Value per year for both the products. Surrender charges decrease and vanish after 6 or 4 years. The main difference between AXA and Metlife products is the way the insurance companies manage the guarantees: AXA declares to use internal synthetic hedging, while Metlife a simpler CPPI strategy.

3. Variable annuities Risk Management and Pricing

There are generally three buckets of risk that exist with almost all life insurance products, Variable Annuities included. These are insurance risk, market risk, behavioral or utilization risk. For Variable Annuities specifically, longevity risk is the primary insurance risk due to the nature of the income guarantees that are offered; some mortality risk exists due to the nature of the death benefit guarantees that are offered. Equity risk and interest rate risk are the primary market risks due to the underlying equity and fixed-income investments that drive the policyholder's account value performance and the long-term nature of the income guarantees. In addition, some credit risk are also present in the fixed-income investments. Persistence risk and benefit utilization risk are the primary behavioral or utilization risks due to the nature of the product structure

that generally has the insurer receiving revenue over time and insurance claims being paid well into the future. Insurers use a number of lines of defense to manage the above buckets of risks. First, they have to design products adopting prudence measures and assumptions. Financial risks are reduced by dynamic hedging that is usually performed daily by the Hedging team of the insurance company. Risk Management team has to evaluate the tightness of the system in the presence of shocks. Tests are conducted regularly, for both the interest of the insurance company and the fulfillment of international norms. Those tests include stress scenario analysis for single and combined shocks and the verification of appropriate provision and management of economic risk capital. Finally, risk pooling (“law of large numbers”), asset liability management (ALM) and reinsurance help in reducing the risks. These lines of defense are employed to varying degrees in an insurer’s risk management strategy, depending on the nature of the risk and the availability and effectiveness of each method. Not all of the lines of defense listed are used with all risks or types of insurance. For instance, reinsurance is generally not used as a primary risk management strategy due to the current limited availability of reinsurance for Variable Annuities guaranteed benefits.

To regulate the management of risks associated with the Variable Annuities, EIOPA decided to create in 2010 a working group dedicated to variable annuities, with the aim of promoting a process of legislative harmonization among the countries of the European Economic Area already during transition from Solvency 2 to Solvency 3. The report published by the European Surveillance in 2011, stressed the importance of drawing a defining perimeter of Variable Annuity policies. In fact, those contracts are not expressly prescribed, and have been approached in a more or less coherent, homogeneous and detailed in the various Member States. Moreover, thanks to their extreme flexibility, they are present on the market through different combinations of possible guarantees. EIOPA insists on a clear and safe definition, by companies, of appropriate financial strategy to the reality of the company, with particular reference to risk appetite. In the chapter devoted to the actuarial profiles, the EIOPA’s recommendations therefore focus on the periodic rebalancing modes of the risk margin. For example, they recommend using simulations and calculations of prospective values of the guarantees and of the underlying assets, but also on the frequency with which to update this evaluation and thus the hedging portfolio as a function of market movements. The particulars provided in the document refer to the hedging programs to be adopted, the calculation methods to be used for technical provisions and to further requirements and controls to be carried out in the course of management.

Among the different phases of the risk management process, the pricing and hedging of guarantees, i.e. of the relevant financial options, should be a major concern for the insurer when designing the contract. Appropriate evaluation techniques need to be developed in order to account on one hand for the interaction between financial and mortality/longevity issues, on the other for the policyholder behavior. Before the 2008 financial crisis, insurers were offering annuities that while generally seen as expensive, offered rich benefits. Some provided lifetime incomes that rose by 6 percent to 7 percent per year, no matter what happened with the underlying investments. The underestimation pricing error came out after the financial crisis and insurance companies offered buybacks to investors.

In recent years, Variable Annuities have attracted the attention of the academic world and the world of finance. Some authors have ventured in the difficult task of developing efficient methods for pricing these products. The article of Bacinello, Millosovich, Olivieri, and Pitacco, (2011) is a point of reference in this field: in this paper they classified main GMxBs Variable Annuities, and they computed and compared contract values and fair fee rates under “static” and “mixed” valuation approaches, via ordinary and least squares Monte Carlo methods, respectively.

Forsyth and Vetzal (2014) performed respectively pricing of GLWB and GMWB Variable Annuities in Black and Scholes framework via PDE methods. They considered both “static” and “dynamic withdrawals”, pricing different versions of the products.

Another research article is that of Yang and Dai (2013) that used a tree based model to price GMWB in Black and Scholes model. They considered both “static withdrawal” and “optimal surrender”.

In a recent research paper of Bacinello, Millosovich, and Montealegre (2016), the authors present in a dynamic programming algorithm for pricing variable annuities with Guaranteed Minimum Withdrawal Benefits (GMWB) under a general Lévy processes framework.

We would also cite the interesting research work of Donnelly, Jaimungal, and Rubisov (2014): the authors consider pricing and Greeks calculation through and Alternating Direction Implicit (ADI) method in the advanced Heston-Hull-White model for a simple static GWMB product (the policyholder cannot chose how much withdraw). They consider a complete stochastic model (stochastic volatility and stochastic interest rate), but they do not consider dynamic withdrawals as real products do.

Finally, Costabile (2015) considers a trinomial tree method for evaluating GMWBs under a regime-switching model.

In two recent research papers Goudenège, Molent, and Zanette (2016) and (Working paper) the authors developed four numerical methods to evaluate GLWB and GMWB policies considering stochastic interest rates (Hull-White model) or stochastic volatility (Heston model). We present here some notes about the most important pricing methods. The pricing methods that are used in those papers are four: two Monte Carlo methods and two PDE methods. Two of these methods are “Hybrid” as they combine trees and Monte Carlo simulations or trees and PDE. Before presenting a summary of these four methods, it is worth spending a few words on the trees used.

Given the long maturity of the products studied, the classic trees presented in Nelson and Ramaswamy (1990) or Appolloni, Caramellino and Zanette (2014) are not suitable to discretize the stochastic volatility and the stochastic interest rate. The authors therefore introduced new quadrinomial trees to implement hybrid methods. They built these trees with the aim to combine exactly the first three moments of the associated processes.

Briani, Caramellino and Zanette (2015) introduced the Hybrid Monte Carlo method. The method involves the use of a tree to define a Markov chain for the volatility (respectively the interest rate): using a discrete variable distributed according to the transition probabilities of the Markov chain, a discrete process \bar{v} (respectively \bar{r}) is defined in order to approximate the volatility (respectively the interest rate). The process moves from the root through the tree, describing the scenario for the volatility or the interest rate. The values of the underlying at each time step can be obtained using a simple Euler scheme.

The Standard Monte Carlo method that is used by the authors is defined using the best methods to generate scenarios in the two models considered. In the case of the Heston model, the reference method is a third order scheme by Alfonsi (2010). For the Black-Scholes Hull-White model, the authors have made reference to an exact discretization scheme presented in Ostrovski (2013).

Briani, Caramellino and Zanette (2015) and (2016) presented the Hybrid PDE method. The method uses a tree to define a Markov chain for the volatility (the interest rate). Then, a one variable partial derivative equation (in addition to the time variable) is solved at the tree nodes. To solve the problem of the correlation between the volatility (or the interest rate) and the underlying, an initial transformation is required. For the Heston model it is useful to set, $Y_t^E = \ln(S_t) - \frac{\rho}{\omega} v_t$, where S_t is the value of the underlying fund and v_t its volatility. The Y^E

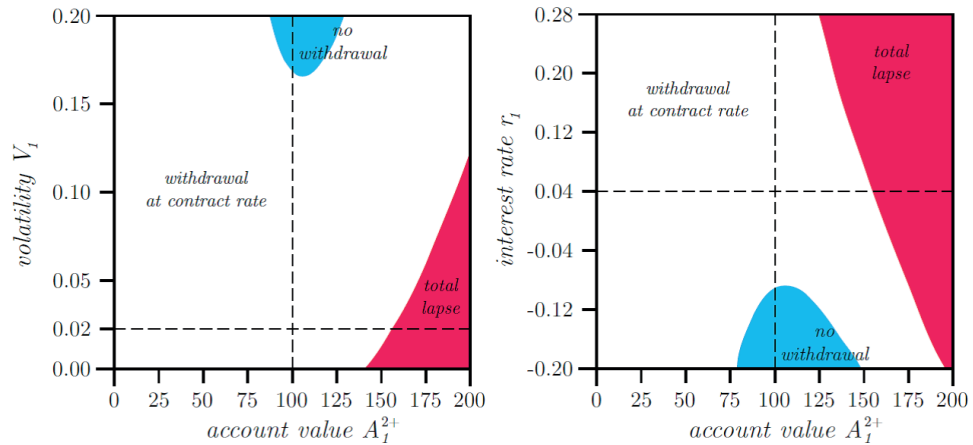
process can be used easily because it is not correlated with the process v . Freezing the value of v at each node, it can be used to define a partial derivative equation (PDE) that is going to be solved by the tree. Similarly, for the Black-Scholes Hull-White we define $Y^U_T = \ln(S_t) - \rho\sigma X_t$ where X_t is a process associated to the interest rate r_t . The Y^U_T process can be used easily because it is not correlated with the process X_t (and therefore with r_t). Freezing the value of X_t (and r_t) at each node, it can be used to define a PDE that is going to be solved by the tree. Hybrid PDE method has proved to be very powerful. It features greater simplicity of coding, stability of results and good speed of convergence.

The ADI method is a method introduced in the 1950s by Peaceman and Rachford (1955) to solve parabolic PDEs. Since then, the method has been used in many sectors. It can be proved that, in the Heston model, the value of an option satisfies a partial derivative equation (PDE) that can be solved using an alternative direction method (ADI).

Such multidimensional equations can be solved using the composition of two schemes that discretize the pair (S,v) (respectively (S,r)) implicitly w.r.t a variable and explicitly to the other. In particular, for Variable Annuities evaluation, the authors used the Douglas scheme with $\theta = \frac{1}{2}$.

The most difficult factor to consider when pricing a Variable Annuity is to determine the optimal withdrawal that the policyholder will choose to perform. In fact, taking the maximum guaranteed amount is not always the best choice: sometimes is more convenient to avoid the withdrawal, and sometimes to terminate the contract (total lapse). This is an optimal control problem. When using Monte Carlo pricing techniques this problem can be faced proceeding backward and applying Longstaff-Schwartz (2001) method. PDE methods naturally proceed backward and the choice of the optimal withdrawal can be faced simply comparing the value of the remaining assets after a given withdrawal. Figure 2 shows an example of this, where the optimal withdrawals for a GLWB Variable Annuity are given.

Figure 2 Optimal strategy of withdrawal for a 66 years old GLWB holder at the first event time $t=1$



All the four numerical methods have been implemented and used to calculate the fair value of the parameter α_{tot} , that express the amount of fees withdrawn by the insurance company from the holder's account. The various numerical tests focused on different variations of both products and they have been done fixing the computational time for all methods: working with a fixed time, it was possible to compare the quality of the results obtained by the different methods.

For both policies GLWB and GMWB, in the static case, the results were very good with all methods: the values obtained are consistent and they differ from each other in small relative

differences. Things have been rather different in the dynamic case. Monte Carlo methods have suffered from the problem of least squares regression: the difficulty of approximating accurately the value function in multiple dimensions has meant that the values obtained by Monte Carlo methods were lower than those obtained with the PDE methods (the withdrawal strategy deduced by the Monte Carlo methods is not fully optimal). These latter, however, proved to be more stable and efficient to address problems of optimal withdrawal. In particular, the Hybrid PDE method provided, in general, the best results.

Conclusion

In this paper, we provided an overlook about Variable Annuities: these interesting insurance products are available to customers of the most financially developed countries and in the last years, they began to be diffused also in new markets, such as China and India. For many investors they represent a valid solution for the retirement problems and it may be same for the new eastern middle class.

We presented some examples from the international markets: the assortment of these products and the different additional features offer to customers a broad range of possible choices attracting their attention.

We also outlined the pricing and hedging principles for these products. These products often have long maturity and researchers continue to work to find effective methods to evaluate these contracts.

This area of research involves problems of different nature: mathematical, probabilistic, numerical, financial, actuarial, stochastic control, social in addition to regulatory issues.

It seems clear that Variable Annuities represent for the coming years an area of research and development, theoretical and applicative.

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